Flame Spread Over Combustible Surfaces for Laminar Flow Systems
Part II: Flame Heights and Fire Spread Rates

K. ANNAMALAI and M. SIBULKIN Division of Engineering, Brown University, Providence, RI 02912
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Abstract—In the previous paper (Part I), results were given for unburnt fuel fraction and wall heat flux for boundary layer flows with combustion. In the present paper, a highly simplified model of the combusting plume region and the previous unburnt fuel results are used to obtain analytical results for the ratio of flame to gasification heights in terms of the parameters \( r \) and \( B \). It is found that these analytical results can be correlated by a simple formula which depends only on the ratio \( r/B \). These results are compared with the predictions of other theoretical work and with available experimental measurements. The flame height results and the heat flux results from Part I are then combined with a simplified physical model to obtain an explicit result for \( \hat{q}_w \) for laminar flame spread on thermally thick surfaces. Using this explicit result the effects of varying parameters such as ambient oxygen concentration, heat of combustion and heat of pyrolysis on the flame spread rate are described. The importance of choosing proper initial conditions when comparing experimental data with theory is discussed, and comparisons with measurements for upward burning on vertical walls are given.

1. INTRODUCTION

The literature on the theory of flame spreading and on flame spread measurements was briefly reviewed in Part I (Annamalai and Sibulkin, 1979). For upward flame spreading on a thermally thick wall (cf., Part I, Figure 1a), heat transfer from the plume above the gasification region to the virgin fuel surface is responsible for the observed rapid flame spread rates. The heat flux in the "combusting plume" region is large compared to the heat flux beyond the flame tip in the "thermal plume" region. Hence the approximation has been made that the heat flux to the unburnt fuel can be assumed to come entirely from the combusting plume region (Orloff, deRis and Markstein, 1975; Fernandez-Pello, 1977). When this approximation is made, the preheat time depends on the heat transfer rate in the combusting plume region \( \hat{q}_w \) and the length of this region \( x_p = x_q \) which, in turn, is dependent on the excess fuel fraction \( F_e \).

In Part I an integral analysis was developed using a normalized coupling function \( \phi(t) \). This analysis led to "integral solutions" for \( F_e \) and \( \hat{q}_w \) (see Part I, Figures 4 and 5) which were obtained by numerical evaluation of the results of the analysis. In addition, it was found that the results of the integral analysis could be reasonably approximated by simple, algebraic "correlations". In Part II, the correlation for \( F_e \) is used to find an approximate result for the flame height, and then the correlation for \( \hat{q}_w \) is subsequently used in the derivation of an algebraic formula for the flame spread rate for upward burning on a thermally thick wall.

2. FLAME HEIGHTS

The excess fuel beyond the gasification region continues to burn in the downstream region. As the excess fuel continues to burn, the fuel fraction near the surface decreases. At the same time the absence of mass transfer from the surface causes the movement of the thin flame towards the surface thus increasing the heat flux. In addition, the surface temperature is not constant. Thus the boundary layer is not self similar in the combusting plume region (cf., Figure 1 of Part I). However, in order to obtain an estimate for the flame height, we have assumed as simplifications that (i) the burning rate for \( x \leq x_q \) continues at the same "trend" as before \( x = x_q \) and (ii) all the excess fuel is burnt. We will then check our results with experimental results as well as with other theoretical results based on more exact theories. Now,

\[
\hat{m}_{F_e} = \frac{F_e \hat{m}_{F_{10}}}{x_q} = \int \hat{m}_{F_{10}} dx.
\]

\[
\hat{m}_{F_{10}} = \left( \frac{x}{x_q} \right) \hat{m}_{F_{10}} \frac{x_q}{y} \frac{y}{x_q} \frac{dx}{y}.
\]

\[
\hat{m}_{F_{10}} = \int \frac{dx}{y}.
\]

\[
\hat{m}_{F_{10}} = \int \frac{dx}{y}.
\]

\[
\hat{m}_{F_{10}} = \int \frac{dx}{y}.
\]

\[
\hat{m}_{F_{10}} = \int \frac{dx}{y}.
\]
For \( x < x_g \),

\[
1 - F_e = \frac{m_{F_e}}{m_{F_e}^*} \frac{1}{n_{F_e}^*}
\]  

(2)

The mass flux \( m_{F_e}^* \) is known for \( x < x_g \); hence, 
burning rate \( n_{F_e}^* \) is known from Eq. (2) for \( x < x_g \). 
Because of assumption (i) and Eq. (2),

\[
\int_{x_g}^{x_f} n_{F_e}^* dx = (1 - F_e) \int_{x_g}^{x_f} m_{F_e}^* dx
\]  

(3)

Rewriting the right-hand side of Eq. (3) as

\[
\int_{x_g}^{x_f} n_{F_e}^* dx = (1 - F_e)
\]

(4)

and then making use of Eq. (1) gives

\[
\frac{m_{F_e}^*}{n_{F_e}^*} \propto x^n.
\]  

(6)

Then, from Eq. (5),

\[
\beta = \left( \frac{x_f}{x_g} \right) = (1 - F_e)^{-1/(n+1)}
\]  

(7)

where

\[n = -1/4, \text{ free convection}\]

\[n = -1/2, \text{ forced convection}\]

\[n = 0, \text{ two-dimensional stagnation flow}\]

\[n = 1, \text{ axisymmetric stagnation flow}\]

Figure 1 presents curves of \( (x_f/x_g) \) vs. \( (1 - F_e) \) for these flow systems. 
In Part I, Eq. (33), we have shown that

\[
1 - F_e \approx 1.4 \sqrt{r/B}
\]  

(8a)

and thus

\[
\beta = \frac{x_f}{x_g} \approx 0.64(r/B)^{-2/3}, \text{ free convection}\]

\[
x_f/x_g \approx 0.51(r/B)^{-1}, \text{ forced convection}\]

\[
x_f/x_g \approx 0.71(r/B)^{-1/2}, \text{ two-dimensional stagnation flow}\]

\[
x_f/x_g \approx 0.85(r/B)^{-1/3}, \text{ axisymmetric stagnation flow}\]

Calculations of flame heights have been given by other investigators. Fernandez-Pello (1977) also made approximations (i) and (ii) above, but calculated numerical similarity solutions in the combusting plume region. Pagni and Shih (1977) used the integral technique in the combusting plume region to convert the boundary layer equations to ordinary differential equations. These equations were then solved numerically. Ahmad et al. (1977) solved the partial differential equations in the combusting plume region by finite difference calculations. (All three calculations use the flame sheet approximation.) Ahmad et al. (1977) also made flame height and heat transfer measurements on a vertical wall for three alcohols using the wick technique.

In Figure 2 we have compared results of these analyses and experiments for free convection on a vertical wall with our simple, explicit result. It appears that use of the parameter \( r/B \) partly correlates results for different \( r \) and \( B \) numbers. (Note...
vertical, combustible surfaces with the assumption of an exponentially decaying heat flux distribution in the preheat region, and obtained results for flame spread velocity for various geometries (rod, slab, semi-infinite solid). Fernandez-Pello (1977) used a similar analysis for spread over a vertical surface, but with a heat flux distribution obtained from a numerical solution assuming local similarity. When local similarity is assumed, the ratio \( x_f/x_0 \) remains constant. Thus the extended flame region \((x_f - x_0)\) grows in time since \(x_0\) increases with time (see Figure 3). Thus it is doubtful whether a quasi-steady state really exists. On the other hand, solving the problem exactly by a numerical approach is unwarranted in view of the need for simpler results. Orloff, deRis and Markstein (1975) carried out an analysis of vertical flame spread velocity for turbulent combustion (or more precisely, for problems with spacewise independent heat flux) which does not involve the quasi-steady assumption. However the heat flux \( \dot{Q}_c \) in laminar flow systems is generally proportional to \( x^n \). Hence we will assume a solution for \( x_0 \) as some function of \( t \) such that the function satisfies the conditions imposed by the physics of the problem and then obtain a solution for the spread of the gasification wave.

3.1 Derivation of the Result

3.1.1 The model. Let the gasification front be at \( Q \) and the flame tip at \( P \) when \( t = t_f \) (Figure 3).

3. FLAME SPREAD VELOCITY

Sibulkin and Kim (1977) developed a quasi-steady analysis for the problem of upward spreading over

---

\[
\frac{r/B}{Q_c/Q_e} \approx \text{const.} \frac{Q_c/Q_e}
\]

Thus the basic fuel property which determines the flame height ratio \( x_f/x_0 \) when Eq. (8b) holds is

\[
Q_c/Q_e
\]

---

\( \dagger \) Values of \( r/B, Q_c/Q_e \) and \( (Q_c/Q_e) \) are tabulated in Table B.1 of Part I.
Consider a time \( t \leq t_1 \). The location \( P \) receives essentially no heat flux from the combustion zone and the temperature of the surface remains at \( T = T_x \). As \( t \to t_1 \), the flame moves to the vicinity of location \( P \); the heat flux \( \dot{q}_{w,0} \) starts rising very rapidly (note that there is no mass transfer and the temperature of the surface increases rapidly. The experimental heat flux distribution is shown qualitatively by the dashed line (Ahmad et al., 1977). For \( t_1 < t < t_2 \) the heat flux remains almost constant and the surface temperature keeps on increasing due to heat flux from the gas phase and internal heat conduction along \( x \). At \( t = t_2 \) the temperature at \( P \) reaches the gasification level \( T = T_g \), and hence the gasification front must be at this location. Now there is mass transfer, and hence the heat flux drops and attain a constant level for \( t > t_2 \). The time \( \tau \) is the actual heat-up time required at the location \( P \) to raise its temperature from \( T_x \) to \( T_g \). Notice that heat flux \( \dot{q}_{w,0} \) does not remain constant within this time \( \tau \).

For modeling purposes, we will assume that the heat flux distribution may be approximated as follows:

\[
\begin{align*}
\dot{q}_{w,0} &= 0, \quad t < t_1 \\
\dot{q}_{w,0} &= \text{constant}, \quad t_1 < t < t_2 \\
\dot{q}_{w,0} &= \text{constant}, \quad t > t_2.
\end{align*}
\]

The solid line in Figure 3 represents the assumed approximate heat flux distribution. Thus the heat-up time with the simplified model is reduced to \( \tau \) rather than \( \tau_0 \).

The time \( \tau \) for heating the location \( P \) from \( T_x \) to \( T_g \) with time invariant heat flux \( \dot{q}_{w,0}(x) \) as shown in Figure 3 can be obtained from the solution of one-dimensional transient heat conduction equation

\[
\frac{\partial T}{\partial t} = \frac{K_x}{\rho C_p x} \frac{\partial^2 T}{\partial x^2}
\]

when it is assumed that longitudinal conduction is negligible compared to the normal heat flux at the surface. For a thermally thick wall the solution of this equation for \( \tau \) is

\[
\tau(x) = \frac{\pi \rho C_p K_x(T_g - T_x)^2}{4[\dot{q}_{w,0}(x)]^2}.
\]

For several types of boundary layer flows

Then

\[
\dot{q}_{w,0} = a_0 x^n.
\]

where

\[
\gamma = \frac{\pi \rho C_p K_x(T_g - T_x)^2}{4a_0^2}
\]

and \( a_0 \) is known from the results in Part 1.

3.1.2 Flame spread velocity. The movement of the gasification wave and flame tip with time is depicted qualitatively in Figure 4. Let us assume that \( x_f \) and \( x_g \) are analytic functions of \( t \). Let \( t_0 \) be any time after initiation of sustained burning (e.g., the minimum value of \( t_0 \) is the ignition time). For \( t > t_0 \) the following three conditions must be satisfied:

i) As seen in Eq. (7), the ratio of flame to pyrolysis heights at any time must remain constant. Mathematically,

\[
x_f(t)/x_g(t) = \beta; \tag{12a}
\]

ii) As shown in Figure 4, at \( t = t + \tau \), the gasification wave location must be at the flame tip location corresponding to time \( t = t \). Thus,

\[
x_g(t + \tau) = x_f(t) = \beta x_g(t) \tag{12b}
\]

where \( \tau \) is a function of \( x \) as given by Eq. (11b) and must correspond to location \( x = x_f \). Thus,

\[
\tau(x_f) = \gamma x_f^{-2n}; \tag{12c}
\]

iii) At time \( t = t_0 \), the gasification wave position is known

\[
x_g = x_g,0; \quad t = t_0. \tag{12d}
\]

![Figure 4 Gasification wave height to flame height relationship as a function of time.](image)
Assume a solution for \( x_g \) of the following form

\[
 t - t_0 = C_1 \{ x_g^{m}(t) - x_{g,0}^{m} \}. \tag{13a}
\]

Then

\[
 t + \tau - t_0 = C_1 \{ x_g^{m}(t + \tau) - x_{g,0}^{m} \}. \tag{13b}
\]

Subtracting (13a) from (13b) and using (12b) for the term \( x_g^{m}(t + \tau) \) gives

\[
 \tau = C_1 x_g^{m}(t) (\beta^m - 1). \tag{14a}
\]

It follows from Eqs. (12c) and (12a) that

\[
 \tau = \gamma \beta^{-2n} x_g^{-2n}(t). \tag{14b}
\]

Comparing (14a) and (14b),

\[
 m = -2n \tag{15a}
\]

\[
 C_1 = \gamma / (1 - \beta^{2n}) \tag{15b}
\]

and therefore, Eq. (13a) becomes

\[
 \{ x_g^{m}(t) - x_{g,0}^{m} \} = [(1 - \beta^{2n}) / \gamma] (t - t_0). \tag{16}
\]

Differentiating with respect to time \( t \),

\[
 V_g \equiv \frac{dx_g}{dt} = \frac{1 - \beta^{2n}}{(-2n)} \frac{x_g^{2n+1}}{\gamma}. \tag{17}
\]

Equations (16) and (17) are true for given arbitrary flow systems as long as the heat flux distribution is given by Eq. (11a) and \( \beta \) can be defined as a ratio of the distance at which \( T = T_\infty \) to the distance at which \( T = T_g \).

If one lets \( n \to 0 \) in Eqs. (16) and (17) for spacewise invariant heat flux conditions and uses L'Hospital's rule, one obtains†

\[
 x_g / x_{g,0} = \beta^{2n} t_0^{1/2} \tag{18}
\]

and

\[
 V_g = x_g \ln \beta / \gamma \tag{19}
\]

which are the results obtained by Orloff, deRis and Markstein (1975) for a turbulent diffusion flame by a different type of analysis.

3.1.3 The importance of initial conditions. If the initial conditions are arbitrarily set as \( x_{g,0} = 0 \) and \( t_0 = 0 \) in Eq. (16), then

\[
 x_g \propto t^{-1/2n}. \tag{19a}
\]

For free convection \( n = -1/4 \), and thus

\[
 x_g \propto t^{1/2}. \tag{19b}
\]

Experiments, however, do not reveal such a law. We will show in Section 3.2 that most of the laminar spread data do fit the results of Eq. (16) when proper initial conditions are used.

3.1.4 Results for free convection. For free convection \( n = -1/4 \). From Eq. (8b)

\[
 \beta = x_f / x_g = 0.64 (r/B)^{-2/3} \tag{20}
\]

and \( \gamma \) is known from Eq. (11c). Using these results in Eq. (16) and Eq. (17) gives

\[
 \{ x_g^{1/2} - x_{g,0}^{1/2} \} = \left[ 4 \left( 1 - 1.15 (r/B)^{1/3} \right) \right] \frac{a_0^2}{\pi} \frac{1}{(\rho c_p K)(T_g - T_\infty)^2} \times (t - t_0) \tag{21}
\]

and

\[
 V_g = \left[ 8 \left( 1 - 1.15 (r/B)^{1/3} \right) \right] \frac{a_0^2}{\pi} \frac{1}{(\rho c_p K)(T_g - T_\infty)^2} x_g^{1/2} \tag{22}
\]

Notice that the ratio \( x_g^{1/2} - x_{g,0}^{1/2} / (t - t_0) \) is known in terms of the properties of fuel and the gas phases.

It is of interest at this point to compare the present result with that obtained by Sibulkin and Kim (1977) where they have used an exponentially decaying heat flux distribution in the preheat region (Appendix A). The result of Sibulkin and Kim is

\[
 \{ x_g^{1/2} - x_{g,0}^{1/2} \} = \left( \frac{2}{3} \phi \frac{Q_f}{Q_\infty} \right) \frac{a_0^2}{\rho_c p K (T_g - T_\infty)^2} \times (t - t_0) \tag{23}
\]

where \( \phi \) is the ratio of heat transferred to the preheat region to the rate of heat release by combustion. Equations (21) and (23) are identical except for the square bracketed terms on the right-hand side. Since \( r/B \approx Q_f / Q_\infty \), one could use these equations to estimate \( \phi \) as a function of \( Q_f / Q_\infty \).

† This result could also be obtained by assuming a solution of the form \( x_g / x_{g,0} = \exp[C(t - t_0)] \) with \( C = \gamma \) and then following a procedure similar to that used to derive Eq. (16).
In order to make explicit the effects of various fuel and gas phase properties on flame spread behavior \(x_p(t)\), Eq. (21) is expanded using Eqs. (39) and (43) of Part I which gives (using \(\rho_k = \text{constant}\))

\[
a_0 = 0.27 \frac{B^{3/4}}{(B+1)^{1/4}} r_{19} Q_p \rho_k w_u (g Q_c / v_u)^{2/3} C_{pc} T_{ao}^{1/4} \frac{Pr}{Pr} \ln(B+1)
\]

Then after rearrangement, Eq. (21) becomes

\[
\begin{align*}
\left( \frac{x_p^{1/2}}{t-t_0} \right)^{1/2} & = \frac{1}{\frac{g^{1/2}}{r_{0.12} B^{1/2}}} \frac{\rho_k C_{pc} K_s}{r_{19} \rho_u C_{pc} K_{uc}} \\
\times \frac{(T_g-T_ao)^{5/2} Y_p T_{pc}^{1/2}}{Y_{o,ao}^{3/2} (Q_c / v_u)^{5/2}} \\
& = 0.073 \left[ \frac{4(1-1.25(r/B)^{1/2})}{\pi} \right] \left[ \frac{Q_p}{Y_{o,ao} (Q_c / v_u)^{5/2}} \right]^2 \\
& \times \frac{r_{0.12} B^{1/2}}{(B+1)^{1/2} \ln(B+1))^2}
\end{align*}
\]

(25)

In order to express the RHS of Eq. (25) in terms of \(r\) and \(B\), the approximation \(B \approx Y_{o,ao} (Q_c / v_u)^{5/2} / Q_p\) is used to give

\[
\begin{align*}
\left( \frac{x_p^{1/2}}{t-t_0} \right)^{1/2} & = \frac{1}{\frac{g^{1/2}}{r_{0.12} B^{1/2}}} \frac{\rho_k C_{pc} K_s}{r_{19} \rho_u C_{pc} K_{uc}} \\
\times \frac{(T_g-T_ao)^{5/2} Y_p T_{pc}^{1/2}}{Y_{o,ao}^{3/2} (Q_c / v_u)^{5/2}} \\
& = G(r, B)
\end{align*}
\]

(26a)

where

\[
G(r, B) = 0.073 \left[ \frac{4(1-1.25(r/B)^{1/2})}{\pi} \right] \frac{r_{0.12} B^{1/2}}{(B+1)^{1/2} \ln(B+1))^2}
\]

(26b)

The function \(G(r, B)\) is plotted in Figure 5 and is seen to have a relatively limited variation for \(0.02 < r < 0.2, 0.5 < B < 10\) which is a realistic range of \(r\) and \(B\) values for \(0.08 < Y_{o,ao} < 0.24\) and the fuels listed in Table B.1 of Part I. As an example of the use of these results, one sees that increasing \(Y_{o,ao}\) corresponds to moving to higher values of \(B\) along a curve of constant \(r/B\) in Figure 5 giving a small decrease in the value of \(G\). If this variation is neglected, Eq. (26) gives

\[
\psi^2 = \left( \frac{x_p^{1/2}}{t-t_0} \right)^{1/2} \propto Y_{o,ao}^3
\]

(27)

showing the large effect of \(Y_{o,ao}\) on \(x_p\). As another example, a decrease in \(Q_c / Q_p\) (a possible result of adding a fire retardant to the condensed phase) corresponds to moving to lower values of \(B\) along a curve of constant \(r\) again leading to a small decrease in \(G\). If the change is due solely to an increase in \(Q_p\), no other change occurs in \(\psi^2\): if the change is due to a decrease in \(Q_c\), then neglecting the change in \(G\) the result from Eq. (26) is

\[
\psi^2 = \left( \frac{x_p^{1/2}}{t-t_0} \right)^{1/2} \propto Q_c^3
\]

(28)

showing the large effect of \(Q_c\) on \(x_p\).

3.1.5 Results for forced convection. For forced convection \(n = -1/2\). From equation (8b)

\[
b = x_p / x_g = 0.51(r/B)^{-1}
\]

(29)

and \(\gamma\) is known from Eq. (11c). Using these results in Eqs. (16) and (17)

\[
\begin{align*}
(x_g - x_g) & = \left[ 4(1-1.96(r/B)) \right] \frac{a_0^2}{\pi} \frac{T_g T_{pc}}{(T_g-T_ao)^2} \\
& \times (t-t_0)
\end{align*}
\]

(30)

and

\[
V_g = \left[ 4(1-1.96(r/B)) \right] \frac{a_0^2}{\pi} \frac{\rho_k C_{pc} K_s (T_g-T_ao)^2}{(T_g-T_ao)^2}
\]

(31)
where from Eqs. (39) and (46c) of Part I

\[
a_0 = 0.26 \frac{B^{1.82} Q_0 p_u (t_{in} x_0)^{1/2}}{P_r^{1.20} (B+1)^{0.44} \ln(B+1)}
\]  

(32)

3.2 Results and Discussion

We concentrate mainly on the quantitative results for buoyant flame spread problems. Using the properties given in Part I for PMMA, we obtain

\[
a_0 = 4.88 \text{ W/cm}^2/\text{s}, \quad x_f/x_g = 3.85
\]

which gives a theoretical value of

\[
\psi = (x_f^{1/2} - x_{g,0}^{1/2})/(t - t_0) = 0.022 \text{ cm}^{1/2}/\text{s}
\]

The corresponding theoretical value (neglecting initial conditions) from Fernandez-Pello (1977) for PMMA is \(\psi = 0.055\). This considerably higher value of \(\psi\) is related to the larger values of \(x_f/x_g\) which he predicts (cf. Figure 2).

Laminar upward flame spread experiments on PMMA slabs were carried out by Orloff et al. (1976) and Fernandez-Pello (1977) where ignition was started with a plane source and hence the flame spread is essentially two-dimensional. Hansen and Sibulkin (1974) ignited PMMA with a point source and hence flame spread is both upward and lateral and the fire growth process is three-dimensional. In order to plot these experimental data in the coordinates suggested by Eq. (21), one needs to know the initial conditions \(x_{g,0}\) and \(t_0\). We plotted all the \(x_g\) vs. \(t\) data on linear graph paper and selected two initial points for each experimental run. Values are given in Table 1; results are shown in Figure 6. Using the coordinates suggested by Eq. (21), the results are found to be nearly linear for both sets of initial points which corresponds to \(\psi = \text{constant}\). The solid lines in Figure 4 are drawn through the points which use \((t_0)\) and \((x_g, t_0)\) as initial data (for clarity, no lines are drawn through the points which use \((t_0)\) and \((x_g, t_0)\) as initial data). The experimental slopes \(\psi_1\) and \(\psi_2\) are given in Table 1. For a given set of data, the maximum difference in \(\psi\) is 11 percent. The dashed line in Figure 6 represents our theoretical result which is roughly twice as high as given by the experiments. Since the theoretical slope depends upon the square of the heat flux in the preheat region it is sensitive to any error in heat flux estimations (such as neglect of surface re-radiation). Thus, the theoretical estimates of heat flux may be too high. In addition, the experimental values of \(\psi\) differ, possibly due to different methods of ignition which affect the preheating of the rest of the sample and different width slabs which may cause nonuniform ignition. More experimental data on flame spread results over other combustible materials are needed to determine the applicability of the simple theoretical results presented in the present paper.

**Table 1**

<table>
<thead>
<tr>
<th>Reference</th>
<th>Symbol in Figure 6</th>
<th>((t_0)) s</th>
<th>((x_g, t_0)) cm</th>
<th>((\phi)) cm(^2)/s</th>
<th>Symbol in Figure 6</th>
<th>((t_0)) s</th>
<th>((x_g, t_0)) cm</th>
<th>((\phi)) cm(^2)/s</th>
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<td>5.00</td>
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</table>
NOMENCLATURE

\[ B = \frac{Q_e \gamma P, \tau}{C_p (T_\infty - T_\infty)} \]

\( C_p \) Specific heat

\( F_r \) Excess fuel fraction, \( \eta_{P, t} / \eta_{F, w} \)

\( g \) Gravitational constant

\( K \) Thermal conductivity

\( m \) Mass transfer rate

\( n \) Similarity exponent, \( n = -1/4 \) for free convection and \( n = -1/2 \) for forced convection

\( Pr \) Prandtl number

\( Q_e \) Heat of combustion

\( \dot{q} \) Heat transfer rate

\( \dot{q}_{w, 0} \) Heat transfer rate to walls under zero mass flux

\( r \) Stoichiometric parameter, \( Y_e / X_F \)

\( T \) Temperature

\( t \) Time

\( u_{\infty} \) Free stream velocity

\( V \) Velocity of gasification wave

\( X \) Distance in flow direction

\( Y \) Mass fraction

\( Y_F, T \) Mass fraction of fuel in transferred gas

Greek Symbols

\( \alpha_0 \) Equation (11a)

\( \beta \) \( x_F / x_0 \)

\( \gamma \) Equations (11b) and (11c)

\( \nu \) Kinematic viscosity

\( \nu_e \) Stoichiometric oxygen–fuel mass ratio

\( \rho \) Density

\( \tau \) Heat-up time to pyrolysis temperature, Eq. (10)

\( \phi \) Ratio of heat transferred to the preheat region to the total heat released in the combusting region

\( \phi \) Equation (27)

Subscripts

\( e \) Excess

\( F \) Fuel

\( f \) Flame

\( g \) Gasification

\( O \) Oxidizer

\( s \) Solid

\( w \) Wall

\( \infty \) Ambient

\( 0 \) Initial condition

Superscripts

' Per unit length

* Per unit area

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REFERENCES


Appendix A

In Sibulkin and Kim (1977) a quasi-steady analysis for upward burning on a thermally thick wall gave (Eq. 34 in the same reference)

\[ V(x) = \frac{\phi \dot{q}_{c, w'} \dot{q}_{w'}^{*}}{\rho_s C_p k_s (T_p - T_{\infty})^2} \]  (A-1)

where \( \dot{q}_{c, w'} \) is the heat released in the whole combusting regime. Using the relationships

\[ \dot{q}_{c, w'} = m \dot{Q}_c \]

\[ \left( \frac{4}{3} \right) \left( \frac{Q_c}{Q_r} \right) \dot{q}_{w'} x_q \]  (A-2)

in Eq. (A-1) yields

\[ V(x) = \left[ \frac{4}{3} \left( \frac{Q_c}{Q_r} \right) \right] \frac{\dot{q}_{w'}^{*2}}{\rho_s C_p k_s (T_p - T_{\infty})^2} \]  (A-3)

where \( j = 1 \) for laminar heat transfer and \( j = 0 \) for turbulent heat transfer. For laminar free convecive burning, \( j = 1 \) and \( \dot{q}_{w'}^{*} = a_0 x_q^{-1/4} \).

Then

\[ V(x) = \left[ \frac{4}{3} \left( \frac{Q_c}{Q_r} \right) \right] \frac{a_0^2}{(\rho_s C_p k_s (T_p - T_{\infty})^2)^{1/2}} x_q^{1/2} \]  (A-4)

Since

\[ t - t_0 = \int_{x_q}^{x_q} \frac{dx_q}{V(x)} \]  (A-5)

one obtains

\[ (x_q - x_q, 0)^{1/2} = \left[ \frac{2}{3} \left( \frac{Q_c}{Q_r} \right) \right] \frac{a_0^2}{(\rho_s C_p k_s (T_p - T_{\infty})^2)^{1/2}} (t - t_0) \]  (A-6)

Comparisons of (A-4) with (22) and (A-6) with (21) show that the expressions differ only by the factors in the square brackets.