Flame Spread Over Combustible Surfaces for Laminar Flow Systems
Part I: Excess Fuel and Heat Flux

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Abstract—The fire hazard of a combustible material may be ranked according to its ignition and flame spread characteristics. For chemically reacting flow systems, the spread rate assumes a special significance because the rate is usually faster than for stagnant systems. What fuel parameters significantly control the flame spread rate? In this paper, we focus our attention on the production of "excess fuel," "excess flame length" due to burning of excess fuel, the heat fluxes to the burnt region and unburnt regions which raise the temperature of virgin surface to the gasification level, and finally the flame spread rate. In Part I, we have used integral techniques to obtain simple results for excess fuel and heat fluxes to the burning surface for buoyant, forced convection and stagnation flow systems; explicit but approximate results are provided for each parameter of interest and they are compared with numerical results and with experiments to provide a check on the present results. In Part II, these results are utilized to obtain approximate but explicit expressions for excess flame length and flame spread rate in terms of known fuel and oxidizer properties.

1. INTRODUCTION

The fire hazard of a combustible material depends upon many parameters such as: (i) initial flaming time, (ii) ignition time, (iii) flame spread rate, (iv) fuel contribution, (v) radiative emission. When the material is burning in a vertical configuration, flame spread rate assumes a special significance because it is most rapid in this orientation, Annamalai and Sibulkin (1976). This paper presents a theoretical analysis of the flame spread process using the concept of the "excess pyrolysis" of Pagni and Shih (1977) and a fire growth model. The "excess fuel" concept, as used here, is generalized to include any material, liquid or solid, undergoing a gasification process (not necessarily pyrolysis alone) and is applied to buoyant, forced and stagnation flow problems. Application of the present technique to flame spread over liquid fuels is limited because of the assumption of negligible convective current inside the liquid medium.

2. REVIEW OF PAST WORKS

Work on flame spreading over surfaces prior to 1968 has been summarized in a review by Friedman (1968). The data are not extensive and much of it is in unpublished company reports. Progress since that time has been surveyed by Friedman (1971) and McAlevy and Magee (1971). McAlevy and his co-workers (1969, 1971) measured flame propagation rates for horizontal, plastic fuel specimens mounted on a metal base plate as a function of ambient pressure and oxygen concentration. Effect of ambient temperature on flame spread process was studied by Perrins and Pettett (1974).

de Ris (1969) presented results for flame propagation rate for thin fuels in terms of thermophysical properties of the fuel and oxidizer and the results seem to be appropriate for vertically downward burning. This theory has been tested by Lastrino et al. (1971) and Sibulkin et al. (1976). Kashiwagi (1975) measured the heat flux and propagation rates for flame spreading over carpet-covered floors, and studied the effects of an imposed radiant flux. Williams and co-workers (1973, 1975) have proposed a thermal theory of flame spreading in which the thermal runaway condition of a gas phase reaction of PMMA vapor provides a criterion for determining the flame spreading velocity; this model assumes a special significance when fire suppressants are added to the stream, since suppressants are expected to inhibit gas phase reaction. Sibulkin and his co-workers (1974, 1975, 1976, 1977) studied flame spreading over cylinders and thin and thick sheets as well as presenting a flame spread theory. They use a "constant preheat distance" concept with exponentially decaying heat flux distribution within this distance in conjunction.
with a quasi-steady solid phase conservation equation for energy and obtain an expression for flame spread velocity applicable to thin and thick sheets and rods.

Using essentially the same quasi-steady conservation equation for the flame spread process under buoyant conditions, but with a heat flux distribution obtained from numerical techniques, Fernandez-Pello (1977) analyzed the problem of flame propagation over vertical surfaces with external radiation for thick and thin solids. Orloff, de Riso and Markstein (1975) have proposed a "leapfrogging" concept, for upward turbulent flame propagation. The presumption is that the time to raise the local surface element to gasification or pyrolysis temperature is much larger than the time to ignite the gases produced from the same region; thus such a theory does not show any dependence on gas phase ignition mechanism which is likely to be affected in the presence of fire suppressants. Very little work is available on cellular plastics. Roberts (1971) compared ignition and flame spread for polyurethane foam and wood and analyzed the results using inert heating models.

3. SOME DEFINITIONS

Enveloped flame. The flame completely surrounds the fuel producing region, e.g., diffusion flame around a spherical droplet, under ventilated Burk-Schumann flame.

Nonenveloped flame. Generally an open flame which does not completely surround the fuel, e.g., flame over a vertical surface. See Figure 1.

Excess fuel fraction \( (F_e) \). For nonenveloped flames of those types shown in Figure 1, the fuel \( m_{F, w} \) produced within the gasification region \( (x \leq x_f) \) is not completely consumed in the flame sheet within \( x \leq x_f \). Thus the amount of fuel \( m_{F, e} \) flowing at \( x = x_f \) is defined as excess fuel

\[
F_e = \frac{m_{F, e}}{m_{F, w}}
\]

Burnt fuel fraction. The fuel which is burnt at the flame location up to \( x = x_f \) expressed as a fraction of the total fuel produced up to the gasification wave \( x = x_f \)

\[
1 = F_e = \frac{m_{F, e}}{m_{F, w}} = \int_{x_0}^{x_f} \frac{m_{F, e}}{m_{F, w}} \, dx
\]

Excess flame length \( (x_f - x_0) \). The excess fuel flowing at \( x = x_0 \) may or may not continue to burn depending upon the availability of oxygen. If it burns at least partially, then one has a flame height \( x_f \) greater than \( x_0 \). This extra flame height \( (x_f - x_0) \) is called excess flame length. For free convection, this is also called the combusting plume region.

Preheat region \( (x_c - x_0) \). The region within which the solid temperature varies from \( T_0 \) at \( x = x_0 \) to \( T_c \) at \( x = x_c \) due to both conduction within the solid and heat flux from hot gases, and to external radiation if present.

The general approach in developing laminar flame spread theory in conjunction with gas phase transport of energy is as follows:
i) Present integral conservation equations for steady state laminar momentum, mass and energy transport.

ii) Use polynomial profiles for the conserved quantities to obtain the results for excess fuel and show that the result is approximately independent of the type of flow system.

iii) Use the same profiles to obtain heat flux to the burning surface via the result of mass flux relations. Compare the results with experiments.

iv) Use these relations in Part II in obtaining results for excess flame length and for flame spread velocity. Compare the results with experiment.

4 GENERAL CONCEPT OF EXCESS FUEL

The fuel burnt per unit area at the flame location is less than the gaseous fuel liberation per unit area at the surface (Spalding, 1954a). Thus there is an excess fuel \( n_{ef} \), which is burnt in the excess flame region. Numerical results for such excess fuel in free and forced convection were previously given by Pagni and Shih (1977). Here, by deriving the result for excess fuel in terms of a normalized coupling function (a Schwab–Zeldovich formulation) whose profile in gas phase does not vary significantly from one type of flow to another type (Spalding, 1954b), a general result is obtained for excess fuel for free, forced and stagnation flow systems in terms of measurable fuel parameters viz. the transfer number \( B \) and stoichiometric parameter \( r = Y_0/m_\gamma Y_p/T \).

5. CONSERVATION EQUATIONS

5.1 Assumptions

The following usual assumptions are made: (i) two-dimensional flow, (ii) boundary layer approximation, (iii) Fick’s law, (iv) \( Le = 1.0 \), (v) thin flame model, (vi) \( \rho D \), \( \mu \) and \( \lambda/C_p \) independent of composition, (vii) no viscous dissipation, and (viii) the external flow field such that a similarity solution exists.

5.2 Governing Equations

The conservation equations for two-dimensional or axisymmetric flow are given as follows:

Continuity:

\[
\frac{\partial}{\partial x} (\rho u x) + \frac{\partial}{\partial y} (\rho u y) = 0
\]  
(1a)

where \( k = 0 \) for two-dimensional flows and \( k = 1 \) for axisymmetric stagnation flow (1c)

Momentum:

\[
\rho \frac{\partial u}{\partial x} + \rho \frac{\partial v}{\partial y} = \frac{\partial}{\partial y} \left( \rho \frac{\partial u}{\partial y} + M(x,y) \right)
\]  
(2)

Species:

\[
\rho \frac{\partial Y_i}{\partial x} + \rho \frac{\partial Y_i}{\partial y} = \frac{\partial}{\partial y} \left( \rho D \frac{\partial Y_i}{\partial y} + m_i'' \right)
\]  
(3)

Energy:

\[
\rho \frac{\partial h}{\partial x} + \rho \frac{\partial h}{\partial y} = \frac{\partial}{\partial y} \left( \rho \frac{\partial h}{\partial y} \right) + m_i'' Q_e
\]  
(4)

For one step global chemical reaction, the stoichiometric equation is given as,

\[
F(g) + O(g) = P(g) + q^\circ
\]  
(5)

where

\[
m_i'' = m_i'' I/F
\]  
(6a)

The momentum source \( M(x,y) \) for different flow systems are given by:

\[
M(x,y) = g(\rho_0 - \rho) \text{ free convection over a vertical wall}
\]  
(6b)

\[
= 0 \text{ forced convection over a flat plate}
\]  
(6c)

\[
= \rho \mu_d d\mu dx, \text{ stagnation flow}
\]  
(6d)

For stagnation flows

\[
\phi = ax
\]  

where \( a = u_f/d_{jet} \) for a plate with a finite circular jet

\[
= 4u_f/d_{cy} \text{ for a long cylinder}
\]  

\[
= 3u_f/d_s \text{ for a sphere}
\]  

A brief comment about the value for \( Q_e \) is appropriate. In the Schwab–Zeldovich formulation,
it is implicit in the derivation that the fuel participates in the reaction as a gaseous species at a standard temperature and at a pressure of 1 atm (Williams, 1965; Kosonen and Williams, 1969; Spalding, 1963; Kim et al., 1971). However, heating values and heats of formation are quoted at standard conditions for fuels in their natural states whose phase may not be solid, for example, PMMA in solid state or benzene in liquid state. Thus one has to add the heat of gasification at standard temperature and pressure to the heating values quoted for fuels which are in either liquid or solid phases in their natural states. Since the products also participate in gaseous form, a lower heating value for \( \dot{Q}_c \) must be selected for use in the final results (Spalding, 1963; Williams, 1965).†

Using the Schwab-Zeldovich formulation for species and energy equations (Williams, 1965), one can reduce the set of Eqs. (3) and (4) to two equations,

\[
\frac{\partial \phi}{\partial t} + \frac{\partial \phi}{\partial y} = \frac{\partial}{\partial y} \left( \frac{\partial \phi}{\partial y} \right) + \frac{\rho D}{\rho c} \frac{\partial Y_i}{\partial y} \tag{7}
\]

\[
\frac{\partial \psi}{\partial t} + \frac{\partial \psi}{\partial y} = \frac{\partial}{\partial y} \left( \frac{\partial \psi}{\partial y} \right) + \frac{\rho D}{\rho c} \frac{\partial Y_i}{\partial y} + m^* j \tag{8}
\]

where \( j \) is one of the species selected as the reference species. In Eq. (7) the normalized coupling function \( \phi \) stands for

\[
\phi = \left( \frac{Y_i - Y_i, w}{Y_i} \right) - \left( \frac{Y_i, w}{J} \right) \tag{9a}
\]

or

\[
\phi = \left( \frac{h_{T, w} - Y_i, w}{J} \right) - \left( \frac{h_{T, w}}{Y_i, w} \right) \tag{9b}
\]

Since we have second order differential equations for \( \phi \) with the boundary conditions \( \phi = 1 \) at \( y = 0 \) and \( \phi = 0 \) as \( y \to \infty \)

\[
\phi = \phi_{T, w} \tag{9c}
\]

† However, in the calculation of adiabatic flame temperatures, one must use the lower heat of combustion as obtained from the natural state of the fuel in question.

It has been assumed in deriving Eqs. (7) and (8) that \( Y_i, w \) and \( h_{T, w} \) are not functions of \( x \) which is consistent with the assumption of similarity.

5.3 Transformations from Compressible to Incompressible Flow

In order to convert the simplified conservation equations into incompressible form, let us introduce the following transformations:

\[
\bar{x} = \frac{x^{2k+1}}{2k+1} \tag{10a}
\]

\[
\bar{y} = y^k \int_0^y \frac{\rho}{\rho w} dy \tag{10b}
\]

\[
\bar{u} = u \tag{10c}
\]

With the assumption that \( \rho D = \text{const.} \), use of the transformations given by Eqs. (10) in Eqs. (1), (2) and (7) yields Eqs. (11) to (13). With the additional assumption of a thin flame model, the reaction term in the species conservation Eq. (8) disappears, and hence Eq. (8) reduces to the form shown by Eq. (14) with a discontinuity at \( y = y_f \).

Continuity:

\[
\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{u}}{\partial \bar{y}} = 0 \tag{11}
\]

Momentum:

\[
\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{u}}{\partial \bar{y}} = \frac{\rho u}{\rho w} \frac{\partial \bar{u}}{\partial \bar{y}} + \left( \frac{M(x,y)}{\rho w} \right) \bar{u} \tag{12}
\]

Normalized coupling function:

\[
\bar{u} = \frac{\partial \phi}{\partial \bar{x}} + \frac{\partial \phi}{\partial \bar{y}} = \frac{\phi}{\bar{y}} \frac{\rho w}{\rho w} \frac{\partial \phi}{\partial \bar{y}} \tag{13}
\]

Species:

\[
\frac{\partial Y_i}{\partial \bar{x}} + \frac{\partial Y_i}{\partial \bar{y}} = \frac{\partial}{\partial \bar{y}} \left( \frac{\rho D}{\rho w} \frac{\partial Y_i}{\partial \bar{y}} \right), \quad y \neq y_f \tag{14}
\]

Equations (11) to (14) will be used later to obtain integral equations.

\[
\text{Note:} \quad \bar{u}_c = \frac{a}{(2k+1) \bar{x}} \tag{15}
\]
6. EXCESS FUEL

6.1 Derivation and General Results

Within the same length of gasification region \( x = x_F \),

\[
\text{[Excess Fuel]} = \frac{[\text{Total fuel produced at the surface}]}{[\text{Total fuel burnt at the flame}]} = m_{F,w}' - m_{F,w}'
\]  
(15a)

Let

\[ F_e = \frac{m_{F,w}'}{m_{F,w}} \]  
(15b)

Then, using (15b) in (15a),

\[ F_e = 1 - \frac{m_{F,w}'}{m_{F,w}} \]  
(16)

By definition,

\[
m_{F,w}' = \int_0^{x_F} m_{F,w}'(x)dx \]

\[ m_{F,w}' = \int_0^{x_F} m_{F,w}'(x)dx \]  
(17a)

\[
m_{F,f}' = \int_0^{x_F} m_{F,f}'(x)dx \]  
(17b)

If, and only if, the ratio \( m_{F,f}'/m_{F,w}' \) is independent of \( x \) (it will be proved later that this is generally valid for free, forced and stagnation flow systems satisfying similarity conditions), then using Eqs. (17) in (16),

\[ F_e = 1 - \frac{m_{F,f}'}{m_{F,w}'} \]  
(18)

The burning rate \( m_{F,w}' \) and the production rate \( m_{F,w}' \) can be derived as follows. The interface mass balance yields

\[
m_{F,w}' = m_{w} Y_{F,T} = \frac{m_{F,w}'}{(\rho D \frac{\partial Y_F}{\partial y})_w} \]  
(19a)

\[
\text{and at the flame with a thin flame model}
\]

\[ m_{F,f}' = - \frac{(\rho D \frac{\partial Y_F}{\partial y})_f}{(\rho D \frac{\partial Y_F}{\partial y})_w} \]  
(19b)

Using the transformation for \( y \) from Eqs. (16) in (19a) and (19b), and making use of assumption (vi), Eq. (18) can be shown to be

\[
(1 - F_e) = \left(1 - \frac{Y_{F,w}}{Y_{F,T}}\right) \left(\frac{\partial Y_F/\partial y}{\partial y/\partial y}\right)_w \]  
(20)

For any chemically reacting laminar flow system satisfying similarity conditions and having an endothermic gasification reaction and negligible surface radiation, use of the thin flame model gives (Appendix A)

\[
Y_{F,w}/Y_{F,T} = 1 - \frac{1 + r}{1 + B}
\]  
(21)

where

\[
B = r Y_{F,T} Q_e / Q_{c} - h_{T,w} - h_{T,\infty}
\]  
(22)

\[
r = Y_{F,\infty} / Y_{F,T}
\]  
(23)

\[
Q_{c} = Q_{c} + C_{p}(T_{F} - T_{\infty})
\]  
(23a)

The result (21) is true even for a stagnant spherical diffusion flame. Thus \( Y_{F,w}/Y_{F,T} \) is almost like a property of a fuel under burning conditions. Notice that as \( B \to r \), Eq. (21) gives \( Y_{F,w} Y_{F,T} \to 0 \).

According to the thin flame model, the flame surface where \( Y_F = 0 \) has now moved to the fuel surface. Hence, the minimum value for \( B \) must be at least equal to \( r \): normally, it is much larger.

With \( i = F, j = 0 \) in Eq. (9a), for a thin flame model

\[
\phi = \frac{Y_{F,T} + (Y_{F,\infty}/\gamma)}{Y_{F,w} + (Y_{F,\infty}/\gamma)}
\]  
(24)

Since similarity solutions exist for burning problems in different flow systems (Kosden et al., 1969; Emmons, 1956; Lees, 1956) define

\[
\eta = \frac{y}{\delta}
\]  
(25)

Using Eqs. (21), (24) and (25) in (20), one obtains:

\[
(1 - F_e) = \left(1 + r \right) \left(\frac{\phi_f/\phi_w}{1 + B}\right)
\]  
(26)

where \( \phi_f \) is the derivative with respect to similarity variable \( \eta \). At this juncture, one could go to the conservation Eqs. (11) to (14) and solve for \( \phi_f \) and \( \phi_w \) and obtain excess fuel fraction \( F_e \) or burnt fuel fraction \( 1 - F_e \). An alternate procedure is to assume a polynomial profile for \( \phi \) which satisfies all relevant boundary conditions and then evaluate

\[ X \]  

For the thermally thick case \( T_w \) is the ambient temperature. In other cases, \( T_w \) may be close to \( T_F \) making the second term negligible.
the excess fuel fraction. Not only is the latter procedure simpler, but it also enables one to obtain simple explicit analytical results which then show the dominant group controlling the excess fuel fraction.

### 6.1.1 Polynomial profiles for \( \phi \)

Let

\[
\phi = \sum_{i=0}^{N} a_i \eta^i \quad (27)
\]

The degree of polynomial to be used depends upon the number of known boundary conditions. It is obvious from the governing differential Eq. (13) that for no slip flow the following boundary conditions arise.

At \( y = 0 \), \( \bar{y} = 0 \), \( \eta = 0 \), \( \phi = 1 \) \hspace{1cm} (28a)

and

\[
\bar{y} + D(x) \frac{\partial \phi}{\partial y} = \frac{\bar{y} + D(x) \partial^2 \phi}{\rho \mu^2 \partial \bar{y}^2} \quad (28b)
\]

Using Eqs. (A.4) and (10), Eq. (28b) is rewritten as

\[
\phi_{\infty} = -Bl \phi_{\infty} \quad (28c)
\]

At \( y = \delta \); \( \bar{y} = \delta \); \( \eta = 1 \); \( \phi = 0 \) \hspace{1cm} (28d)

\[
\phi' = 0 \quad (28e)
\]

where Eq. (28f) arises from governing differential Eq. (13). We obtain a third degree polynomial if conditions (28a) to (28e) are used [Spalding, 1954c] and a fourth degree polynomial if condition (28f) is also included. Table 1 lists the coefficients \( a_i \).

These profiles for \( \phi \) may exhibit agreement with numerical results in the neighborhoods of the boundaries where the profiles satisfy appropriate boundary conditions as imposed by the physics of the problem; but the profiles generally do not satisfy the governing differential equation for the whole region \( 0 < \eta < 1 \). Thus the selection of third or fourth degree polynomial for this problem which involves a derivative in the intermediate position of the boundary layer (i.e., \( \eta = \eta_f \)) will be based solely on the agreement of present results for \( F_e \) with numerical results.

Differentiating Eq. (27) and evaluating the derivatives of \( \phi \) at the flame and the surface,

\[
\phi' / \phi = \sum_{i=0}^{N} (a_i / a_0) \eta_f^{i-1} \quad (29)
\]

where the flame location \( \eta_f \) is solved from the equation

\[
\phi_f = \sum_{i=0}^{N} a_i \eta_f^i \quad (30)
\]

Using Eq. (24) at the flame location where \( Y_f = 0 \) and Eq. (21), the nondimensional coupling function at the flame location \( \phi_f \) is found to be

\[
\phi_f = \frac{(1 + 1/B)}{(1 + 1/r)} \quad (31)
\]

Again, note that as \( B \rightarrow r, \phi_f \rightarrow 1.0 \) which is consistent with the result that \( Y_f, w_f \rightarrow 0 \) as \( B \rightarrow r \). The limit \( \phi_f \rightarrow 1.0 \) implies that the flame temperature drops to the surface temperature.

With the use of Eqs. (26), (29), (30) and (31) the following result for excess fuel fraction \( F_e \) is obtained.

\[
F_e = 1 - \frac{1 + r}{1 + B} \sum_{i=0}^{N} (a_i / a_0) \eta_f^{i-1} \quad (32)
\]

where \( \eta_f \) is known from Eq. (30) and \( a_i \)'s are taken from Table 1. Observed that as \( B \rightarrow r, F_e \rightarrow 0 \).

### Table 1

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>( N = 3 )</th>
<th>( N = 4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_0 )</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( a_1 )</td>
<td>(-3/2 \text{ as } B \rightarrow 0, )</td>
<td>(-2 \text{ as } B \rightarrow 0 )</td>
</tr>
<tr>
<td>( a_2 )</td>
<td>(-2a_1 - 3 )</td>
<td>(-3a_1 - 6 )</td>
</tr>
<tr>
<td>( a_3 )</td>
<td>(a_1 + 2 )</td>
<td>(3a_1 + 8 )</td>
</tr>
<tr>
<td>( a_4 )</td>
<td>0</td>
<td>(-a_1 - 3 )</td>
</tr>
</tbody>
</table>

† The flame location computed from the present third degree polynomial appears to yield approximately the same result as the equation \( \eta_f = (1 - \phi_f^{1/2}) \) which was used by Kim et al. (1971).
6.2 Results and Discussion

The values for excess fuel fraction \( F_e \) are computed from Eq. (32) for several \( B \) and \( r \) numbers and are shown in Figure 2. A check on the present results is provided by a comparison with the tabulated numerical results obtained by Pagni and Shih (1977) for a forced convection system in Figure 3. We have also compared the results obtained both from 3rd and 4th degree polynomial profiles for \( \phi \) using the same \( B \) numbers as quoted by Pagni et al. Both polynomials give the same qualitative result, but the results from the 3rd degree polynomial are closer to numerical results. In order to check to what extent the excess fuel fraction depends upon whether the flow is buoyant or forced convection, we have compared the present results with the estimates from the graphical results of Pagni and Shih for free convection (no tabulated results are available). The numerical results do show a higher excess fuel fraction by about 20 percent for free convection compared to forced convection. Thus, for free convection the 4th degree polynomial profile may give closer agreement than the 3rd degree polynomial.

In order to obtain simple approximate results for \( F_e \) and fluxes of heat and mass we use a polynomial of 3rd degree for the excess fuel fraction, heat and mass flux calculations given in the remainder of the paper.

A search for an empirical correlation which would represent the results of Figure 2 in an analytical form and which could be used for further analysis led to the following simple relationship

\[
1 - F_e \approx 1.4 \sqrt{r/B}, \quad B \geq 2r
\]  

Figure 4 plots \( 1 - F_e \) vs. \( r/B \) and also gives the ranges of \( r/B \) for various fuels. A compilation of \( B \) and \( r \) numbers for solid and liquid combustibles is given in Appendix B to aid in the application of this analysis. We note that liquid fuels generally have lower \( r/B \) ratios and hence higher excess fuel fractions than solid fuels.

From the definition of \( B \) number, if sensible enthalpy ratio \((h_T,_{w}-h_{T,\infty})/Q_0 \) is small compared to heat release in combustion \((r Y_F, r Q_F/Q_0) \) then
and hence

\[ B \approx \tau Y_F, T Q_c S \]

(34)

\[ 1 - F_e \approx \frac{1}{2} \sqrt{Q_c^2 Y_F, T} \]

(35)

Thus the result for excess fuel is independent of oxygen concentration when Eq. (34) holds. Physically it is clear that as the heat of combustion per unit mass increases for the same heat of pyrolysis, the flame will stand farther from the surface in order to reduce the thermal gradient (see simple analysis of Spalding, 1952). This larger flame stand-off distance leads to a higher excess fuel fraction.

\[ \dot{q}_w = (\rho u)\dot{Q}_F = \frac{K_w}{C_p} \left( \frac{\partial T}{\partial y} \right)_w \]

(36)

Let

\[ \eta = \frac{y}{\delta_p} \]

(37a)

(37b)

(37c)

\[ \delta_u = \int_0^{\delta_u} \frac{\rho}{\rho_w} x^k dy \]

(37d)

\[ \delta_p = \int_0^{\delta_p} \frac{\rho}{\rho_w} x^k dy \]

(37e)

Using Eq. (9b) with \( j = 0 \) and Eqs. (10b) and (37a)

\[ \dot{q}_w x/\rho_w D_w Q_F = (\rho u)_{\infty} x/\rho_w D_w = -B a_1 x/(\delta_p x) \]

(38)

where the polynomial profile for \( \phi \) has been made use of in obtaining a derivative \( \phi'(x) = \partial \phi/\partial y \) at the surface. Thus the problem is to obtain \( (\delta_p x) \) for different flow systems. Note the polynomial profile for \( \phi \) and hence \( a_1 \) do not change from one flow to another.

7. Heat Fluxes

In order to evaluate the flame spread theory to be developed in Part II, a knowledge of the heat flux distribution in simple explicit form is desirable. The earliest work of this kind seems to be the integral techniques of Spalding who presents results in graphical form based upon assumptions of constant properties and incompressible flow. Emmons (1956) presents numerical results for forced convection with variable properties for \( Pr = Sc = 1.0 \), and recently Tien and Prahll (1977) give numerical results for stagnation flow systems. Kim, deRisi and Markstein (1971) give both numerical and explicit integral results for free convection. Since the Polhausen integral technique is so well known once the equations are converted from compressible to incompressible form, no details of the derivation for the integral solutions will be shown here.

7.1 General Relation

7.1.1 Flux under mass transfer. The heat flux is essentially obtained from the mass flux to the surface. For the burning problem, the heat balance at the surface gives,

\[ \dot{q}_w = (\rho u)\dot{Q}_F = \frac{K_w}{C_p} \left( \frac{\partial T}{\partial y} \right)_w \]

(36)

Let

\[ \eta = \frac{y}{\delta_p} \]

(37a)

\[ \eta' = \frac{y'}{\delta_u} \]

(37b)

\[ \tau = \frac{\delta_u}{\delta_p} \]

(37c)

\[ \delta_u = \int_0^{\delta_u} \frac{\rho}{\rho_w} x^k dy \]

(37d)

\[ \delta_p = \int_0^{\delta_p} \frac{\rho}{\rho_w} x^k dy \]

(37e)

Using Eq. (9b) with \( j = 0 \) and Eqs. (10b) and (37a)

\[ \dot{q}_w x/\rho_w D_w Q_F = (\rho u)_{\infty} x/\rho_w D_w = -B a_1 x/(\delta_p x) \]

(38)

This approximation gives a jump in local heat flux by a constant ratio \( [B/\ln(1+B)] \) downstream of the gasification wave within the excess flame region.
7.2 Free Convection

Using the Pohlhausen technique and the definition of \( f \) given in Table 2, the momentum Eq. (12) and the coupling function Eq. (13) reduce to

\[
\frac{1}{\varepsilon} \int_0^1 \frac{d}{d\eta} \left( \frac{\alpha_{\text{max}} \delta_{\phi}}{\delta_{w}} \right) I_{m,b} + \frac{1}{C_p \varepsilon} \frac{Q_{\xi} Y_{\xi}}{C_{\eta}} I_{\phi,b} = \frac{1}{C_p \varepsilon} \frac{\alpha_{\text{max}} \delta_{\phi}}{\delta_{w}} \left( \frac{\phi_{\text{max}}}{C_p \varepsilon} \right) \tag{40a}
\]

where

\[
I_{m,b} = \int_0^1 \xi d\eta \tag{40b}
\]

(momentum integral for buoyant flow),

\[
H = \int_0^1 \frac{T - T_\infty}{T_\infty} d\eta \left( \frac{Q_{\xi} Y_{\xi}}{C_p \varepsilon} \right) \tag{40c}
\]

(buoyancy integral), and

\[
+ \frac{1}{\delta_{w}} \frac{d}{d\xi} \left( \frac{\varepsilon_{\text{max}} \delta_{\phi}}{\delta_{w}} \right) I_{\phi,b} = \left( B + 1 \right) \phi_{\text{max}} \frac{D_{w}}{\delta_{w}} \tag{40d}
\]

where

\[
I_{\phi,b} = \int_0^1 \phi d\eta \tag{40e}
\]

(coupling function integral for buoyant flow).

The temperature profiles are found from the coupling function profiles using Eqs. (27) and (9b) with \( Y_{\xi} = 0 \) for \( \eta > \eta_0 < \delta_{\phi} \) and \( Y_{\xi} = 0 \) for \( 0 < \eta < \eta_\tau \). Hence \( H \) is known as a function of \( B, r, \tau \) and \( Q_{\xi} Y_{\xi} \varepsilon / C_p T_\infty \). The profile for \( \phi \) is given by Spalding (1954) as

\[
f = \sum_{i=0}^{3} c_i \eta^i \tag{41}
\]

where the coefficients \( c_i \) and the definitions for \( f \) are tabulated in Table 2 for different problems. Thus, using the profiles for \( \phi \) and \( \delta_{\phi} \), one obtains the results for \( \delta_{w}/x \) and \( \delta_{w}/x \) given in Table 2. But since \( \delta_{w} = r \delta_{\phi} \), it is still necessary to solve for \( \tau \) by iteration. However, because \( H, I_{\phi,b} \), and \( \tau \) occur inside the 1/4th power root, approximate quantitative results \( \tau \) can be set equal to unity. Then

\[
I_{m,b} = c_1 T_{1/0.5} \tag{42a}
\]

The nondimensional boundary layer thickness \( \delta_{w}/x \) can now be evaluated from the formula given in Table 2. The heat flux is then known from Eq. (38).

It was found from numerical calculation that the function \( H \) is roughly proportional to \( r \) and independent of \( B \) for \( 0.5 < B < 20 \), and that the coefficient \( (\alpha_{\text{max}}) \) varies as \( 1.28 B^{0.05} \) for \( 0.7 < B < 100 \). Letting \( Q_{\xi} Y_{\xi} \varepsilon / C_p T_{\infty} \approx B/r \) in the \( H \) function, an approximate result for the heat flux is then given by

\[
(q_w)_{\text{approx}} = \left( \frac{Q_{\xi} Y_{\xi} \varepsilon / C_p T_{\infty}}{(B + 1)^{1/5}} \right) \left( \frac{B}{r} \right) \tag{43}
\]

\[= 0.27 \left( \frac{B}{(B + 1)^{1/5}} \right) r^{-0.08}, \quad B \ge r \]

It may be mentioned that the term in square brackets is not far from unity for most fuels (cf., Table B.1).

Heat flux results calculated from the integral solution, from the correlation equation, and from the exact numerical solution of Kim et al. (1971) are compared in Figure 5 where \( \tau \) is \( C_p T_\infty - T_\infty / Q_{\xi} \). The correlation equation gives an excellent fit to the integral solution, and both are in good agreement with the numerical solution in the range \( 1 < B < 7 \). For \( B < 7 \), the burning rate is almost linearly proportional to \( B \) and thus to \( Y_{\xi} \) for a given fuel.

Table 3 presents a comparison between the approximate integral results and experimental heat flux measurements. Good agreement exists for the gasification region, and for the excess flame.

\[\text{We note that the combination } \rho \mu \phi = (1/r)^{1/3} \text{ which occurs in the definition of } (q_w)_{\text{approx}} \text{ equals } (\rho \mu \phi)^{1/2}. \text{ Since it has been assumed that } \rho \mu = \text{ const, we have evaluated } \rho \mu \text{ at the wall.}\]
\[
\begin{align*}
\dot{m}'' &= q_{d} \frac{D_{B}}{\sqrt{Q_{p} / x^{*} + 1}} \times \\
\text{Free Convection} \quad (T_{m, b} / c_{l}^{2}) &= \frac{1}{16} + \frac{G_{1}}{105} \\
\text{Forced Convection} \quad (T_{m, f} / c_{l}^{2}) &= \frac{\tau_{f}}{C_{f}} + \frac{A_{1}^{3}}{A_{f}} (1 + A_{f})^{2} \\
\text{Stagnation flow} \quad u_{*} &= 1.0 \\
\text{Forced convection} \quad (T_{m, f} / c_{l}^{2}) &= \frac{A_{1}^{3}}{A_{f}} (1 + A_{f})^{2} \\
\text{Stagnation flow} \quad u_{*} &= 0.1 \\
\text{Forced convection} \quad (T_{m, f} / c_{l}^{2}) &= \frac{A_{1}^{3}}{A_{f}} (1 + A_{f})^{2} \\
\text{Stagnation flow} \quad u_{*} &= 0.1 \\
\end{align*}
\]
region for the alcohols. In the excess flame region, Spalding's inverse mass transfer correction (Eq. 39) has been used as discussed in Section 7.1.2.

7.3 Forced Convection

The integral equations for momentum and coupling function are

\[ I_{m,t} \frac{d \delta_u}{d \xi} = (-a_1) \frac{D_w \rho R}{u_{\infty}} \left[ \frac{B_r}{Pr} + \frac{C_1}{a_1} \right] \]  (44a)

\[ I_{\phi,t} \frac{d \delta_{\phi}}{d \xi} = (-a_1)(B+1) \frac{D_w}{u_{\infty}} \]  (44b)

where

\[ I_{m,t} = \int_{0}^{1} f(1-f) d\eta \]  (45a)

(momentum integral for forced convection),

\[ I_{\phi,t} = \int_{0}^{1} \phi(1-\phi) d\eta \]  (45b)

(coupling function integral for forced convection).

The results for \( \delta_u/\xi \) and \( \delta_{\phi}/\xi \) are shown in Table 2.

When \( Pr = 1 \) and \( \tau = 1 \)

\[ I_{m,t} = \frac{9}{70} \left( \frac{A_f}{B_r} \right)^2 - \frac{3}{140} C_1 - \frac{1}{16} \]  (46a)

\[ I_{\phi,t} = \frac{9}{70} \left( \frac{13}{420} \right) \frac{A_f}{a_1} \]  (46b)

Notice from Table 2 that \( C_1 = a_1 \) when \( Pr = 1.0 \). Thus one only needs the result for \( \delta_{\phi}/\xi \) under these conditions.

For \( Pr \neq 1 \), the analysis shows that \( \tau \neq 1 \) but varies weakly with \( Pr \) in a complex way. Spalding (1954b) has solved the incompressible flow problem assuming constant properties and \( Pr = 0.71 \). In the range \( 0.5 < B < 10 \), the integral solution plotted in Figure 6 agrees with Spalding's result within 7 percent. The solution is compared with Emmons

<table>
<thead>
<tr>
<th>TABLE 3</th>
</tr>
</thead>
</table>

Comparison of theoretical and experimental results for nondimensional heat flux \( (\dot{q}/\dot{h}) \star \)

<table>
<thead>
<tr>
<th>Fuel</th>
<th>( B )</th>
<th>( B )</th>
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<th>( B )</th>
<th>( B )</th>
<th>( B )</th>
<th>( B )</th>
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<th>( B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Methanol</td>
<td>2.6</td>
<td>0.57</td>
<td>0.62</td>
<td>2.96</td>
<td>0.96-1.18</td>
<td>Free convection</td>
<td>Ahmad et al. (1977)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ethanol</td>
<td>2.9</td>
<td>0.72</td>
<td>0.70</td>
<td>1.65</td>
<td>1.54-1.62</td>
<td>Free convection</td>
<td>Ahmad et al. (1977)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Propanol</td>
<td>3.3</td>
<td>0.78</td>
<td>0.76</td>
<td>1.86</td>
<td>1.79-1.92</td>
<td>Free convection</td>
<td>Ahmad et al. (1977)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PMMA</td>
<td>8.5*</td>
<td>0.60</td>
<td>0.28-0.56b</td>
<td>---</td>
<td>---</td>
<td>Forced convection</td>
<td>Krishnamurthy and Williams (1973)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PMMA</td>
<td>1.8</td>
<td>0.59</td>
<td>0.55-0.67</td>
<td>---</td>
<td>---</td>
<td>Axysymmetic stagnation flow</td>
<td>Holve and Sawyer (1974)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* For the experimental condition \( \gamma_{u,\infty} = 1.0 \).

* The experimental data is taken from Figure 8 in the quoted reference for the region near the leading edge.
where

\[ I_{m,s} = \int_0^1 f(1-f) d\eta' \]  
(48a)

(momentum integral for stagnation flow)

\[ I_{pfr,s} = \int_0^1 \left( \frac{\rho_0}{\rho} - 1 + f \right) d\eta' \]  
(48b)

\[ I_{\phi,s} = \int_0^1 (1-f) \phi d\eta \]  
(48c)

(coupling function integral for stagnation flows).

The results for \( \delta_{\eta}/\bar{x} \) and \( \delta_{\phi}/\bar{x} \) are tabulated in Table 2. For stagnation flows, because of the pressure gradient term, the use of \( \tau = 1 \) is not a good approximation. Hence, using \( \delta_{\eta}/x \) and \( \delta_{\phi}/x \), \( \tau \) was solved from the set of complex algebraic equations. For \( B \) varying from 0.1 to 20.0, \( \phi \) ranging from 0.01 to 2.0, \( \rho_0/\rho \) varying from 1.0 to 5.0, and \( Q_e/C_p T_{\infty} \) varying from 80 to 40, the nondimensional heat

\[ \dot{q}_w = q_w^* x Pr^{1/3} Q_e p_a v_\infty \sqrt{Re} \approx 0.26 \left( \frac{B}{\sqrt{B+1}} \right)^{0.82} \]  
(46c)

for \( B > r \)

As shown in Figure 6, the error is almost negligible. Table 3 compares the present result with one experimental result.

7.4 Stagnation Flows

The integral equations for momentum and coupling function are

\[ \frac{I_{m,s}}{\bar{u}_s^2} \frac{d}{d\bar{x}} [\bar{u}_s \delta_\eta^2] + I_{pfr,s} \frac{d \bar{u}_s}{d\bar{x}} = \left( -a_1 \right) \frac{\rho_0^2 D_w Pr \tau}{\delta_\eta^2} \]  
(47a)

\[ I_{\phi,s} \delta_\phi \frac{d}{d\bar{x}} [\bar{u}_s \delta_\phi] = \left( -a_1 \right) \frac{\rho_0^2 D_w [B+1]}{\delta_\eta^2} \]  
(47b)
FLAME SPREAD OVER SURFACES: PART I

Fluxes were computed, giving the results shown in Figure 7. A simple correlation of these results (for $0.5 < r < 10$) is given by

$$\dot{q}_w^* = \frac{\dot{q}_w^* x Pr^{1/2}}{Q_{fpw} x [Re_s(k - 1)]^{1/2}} \approx 0.60 \left( \frac{B}{\sqrt{B+1}} \right)^{0.84} r^{0.04}$$

for $B > r$ (49)

For PMMA, using $B = 1.44$, $r = 0.12$ (as used by Tien et al., 1976), the numerical solution for axisymmetric stagnation flow is $\dot{q}_w^* = 0.64$, while the integral results for the same diffusion limited condition is $0.51$, thus yielding an error of about 20 percent. This is the price one has to pay when an overall correlation group is more desirable. Comparison with one experimental result (Table 3) for PMMA gives fairly good agreement.

ACKNOWLEDGEMENT

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NOMENCLATURE

- $a$: a constant in stagnation flows, $u_e = ax$
- $a_t$: Eq. (27) and Table 1
- $B$: transfer number, Eq. (22)
- $C_i$: Eq. (39) and Table 2
- $C_p$: specific heat at constant pressure
- $D$: diffusion coefficient
- $d$: diameter
- $F_i$: excess fuel fraction
- $Gr$: Grashoff number, $(g x^2 Q_g)^{1/2} C_p T_m$
- $g$: gravitational constant
- $H$: Eqs. (38c) and (40b), also refer to Table 2
- $h$: enthalpy
- $i, j$ etc.: stoichiometric mass of species, $i, j$ etc., participating in reaction
- $K$: thermal conductivity
- $k$: = 0 for 2 dimensional flows
- $= 1$ for axisymmetric stagnation flow
- $Le$: Lewis number, $N_p D C_p$
- $M$: momentum sources in laminar flows, Eqs. (2), (6b), (6c) and (6d)
- $m$: mass flow
- $N$: maximum order of polynomial for $\phi$
- $\Phi$: heat of combustion of fuel
- $\Phi_{fu}$: heat of gasification at gasification temperature
- $\Phi_f$: heat required to gasify unit fuel mass which is initially at standard temperature of 25°C
- $q^*$: heat of reaction, Eq. (5)
- $\bar{q}^*$: nondimensional heat flow
- $r$: temperature
- $u$: velocity along $x$
- $v$: velocity along $y$
- $x, y$: see Figure 1
- $Y$: mass fraction
- $Y_F, T$: mass fraction of fuel in the transferred phase

Greek Symbols

- $\delta$: boundary layer thickness
- $\xi$: similarity variable ($= y/\delta_\infty$
- $\eta$: similarity variable ($= y/\delta_\infty$
- $\mu$: absolute viscosity
- $\nu$: kinematic viscosity
- $\nu_t$: stoichiometric oxygen–fuel mass ratio
- $\rho$: density
- $\tau$: ratio of momentum boundary layer thickness to coupling function boundary layer thickness ($= \delta_\infty/\delta_\rho$
- $\phi$: normalized coupling function, Eq. (9)

Subscripts

- $e$: excess when associated with fuel, edge when associated with velocity, external
- $f$: flame
- $g$: gasification
- $i$: species
- $m$: momentum
- $s$: solid
- $T$: thermal
- $u$: velocity layer
- $w$: wall or surface
- $0$: zero mass transfer
- $\infty$: ambient
- $\phi$: coupling function

Superscripts

- $0$: standard condition
- $/'$: per unit width
- "$/": per unit area
- "$/$": per unit volume
- $^\prime$: variable in incompressible coordinates
REFERENCES


Appendix A

DERIVATION OF FUEL FRACTION AT THE SURFACE

Equation (9b) with $j = 0$ gives

$$
\phi = \frac{h_T - h_{T,\infty}}{Q_c} - \frac{Y_f}{v_2} \quad 0 < \eta < \eta_f \quad (A-1)
$$

and with $j = F$ gives

$$
\phi = \frac{h_T - h_{T,\infty}}{Q_c} + \frac{Y_f}{v_2} \quad 0 < \eta < \eta_f \quad (A-2)
$$

Satisfying the energy balance at the gasification surface yields

$$
\frac{q_w}{C_p} \left( \frac{\partial y}{\partial y} \right)_w = m_w Q_g \quad (A-3)
$$

Using Eq. (A-1) and definition of $B$ number, the Eq. (A-3) can be written

$$
m_w = (\rho v) w = - \frac{B K_w}{C_p} \left( \frac{\partial \phi}{\partial y} \right)_w \quad (A-4)
$$

Differentiate both Eqs. (A-1) and (A-2) with respect to $y$ and obtain $\phi, \partial \phi/\partial y$ in each case, equate them to obtain $\partial (\partial y) / \partial y$; then use Eq. (19a) to eliminate $\partial \phi/\partial y$ and Eq. (A-3) to eliminate $\partial ^2 y / \partial y ^2$ to finally obtain $Y_f, v_2, Y_f, v_2$ as given by Eq. (21). Note that we have not made use of any fluid-dynamic characteristics associated with a flow, but similarity conditions must be satisfied so that one has an ordinary differential equation in $\phi$ which has two known boundary conditions, $\phi = 1$ at the surface and $\phi = 0$ far from the surface.
<table>
<thead>
<tr>
<th>Fuel Material</th>
<th>Formula</th>
<th>T_0 [K]</th>
<th>KJ/Kg</th>
<th>KJ/Kg</th>
<th>C_4H_10</th>
<th>C_3H_6</th>
<th>B</th>
<th>T</th>
<th>G/Q</th>
<th>C_P or C_m</th>
<th>V/Kg/m³</th>
<th>k or K</th>
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<td>2320°</td>
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<td>0.0446</td>
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<td>1.178</td>
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<tr>
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<td>2203°</td>
<td>2320°</td>
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<td>0.3</td>
<td>1300</td>
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<td>1.2</td>
<td>0.052</td>
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<td>1030°</td>
<td>2780°</td>
<td>0.2</td>
<td>1335</td>
<td>0.150</td>
<td>2.5</td>
<td>0.069</td>
<td>0.0596</td>
<td>792°</td>
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<tr>
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<td>792°</td>
<td>2.37</td>
<td>0.202</td>
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<tr>
<td>Ethane</td>
<td>C_3H_6O</td>
<td>871°</td>
<td>1030°</td>
<td>2780°</td>
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<td>C_2H_6O</td>
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<td>1030°</td>
<td>2780°</td>
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<td>2780°</td>
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<td>2780°</td>
<td>0.2</td>
<td>1335</td>
<td>0.150</td>
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<td>0.0596</td>
<td>792°</td>
<td>2.37</td>
<td>0.202</td>
</tr>
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</table>
Appendix B

Tabulation of Fuel Properties

In this appendix, property values for polymers, alcohols and liquid fuels were collected from literature and are tabulated. Table B.1 tabulates thermophysical and chemical properties of some solid and liquid combustibles. In part [A] we have tabulated B and r numbers and other properties necessary to calculate B-number from basic definitions as given by Eqs. (22) to (23a). For the calculation of B-numbers in air, values for $T_\beta$, $Q_\beta$, $Q_\delta$ and $v_\beta$ must be known. The temperature of gasification of some polymers have been previously measured during combustion and are reported in Table B.1. For a few other polymers, the values of $T_\beta$ were taken from experimental kinetics data on mass loss vs. temperature, since most of mass loss occurs within a narrow temperature range. Such values are indicated in brackets ( ). For liquid fuels the gasification is purely physical and as such $T_\beta$ is known from thermodynamic data.

The heat of gasification $Q_\beta$ includes the sensible heat to raise the temperature from 25°C to the gasification temperature. This value is useful in studying the burning of a semi-infinite body of a solid or liquid. However, for studying thin solid, thin liquid film or liquid droplet combustion, the value of $Q_\beta$ at the gasification temperature must be used.

The heat of combustion $Q_\epsilon$ is the lower heating value for a gaseous fuel. The data for the polymers (in a virtual gaseous state) was calculated from previously published heating values for solid polymers in their natural states. Thus for polymers

\[ Q_\epsilon = Q_\epsilon^0 + Q_\beta + h_{r,5} - h_{r,5}^{ref} \]

\[ Q_\epsilon \approx Q_\epsilon^0 + Q_\beta + (C_{pr} - C_{pr}^{ref})(T_\beta - T_\alpha) \]

where $Q_\epsilon^0$ is the lower heating value for the fuel in its natural state where $C_{pr}$ is the specific heat of the polymer gas.

The stoichiometric oxygen requirement $v_\beta$ per unit mass of fuel for combustion to CO$_2$ and H$_2$O is known from the chemical formula of fuel of interest. For polymers the tabulation is arranged in the order of increasing values for $r$. For the B-number calculation, when data on $T_\beta$ for polymers was not available, it is assumed that $T_\beta \approx 600$ K for calculation purposes and the results are indicated with an asterisk.

The ratios $r/B$ and $Q_\delta/Q_\epsilon$ were tabulated to indicate degree of error involved in using

\[ r/B \approx Q_\delta/Q_\epsilon \]

when $Y_\beta, T = 1$. The ratio $Q_\delta/v_\beta$ was tabulated to show the degree to which the heat of combustion per unit mass of oxygen is independent of the type of fuel.

In part [B] of the Table B.1, we have tabulated $p$, $C_p$ and $K$ of the fuels in their condensed state. This is useful if one is interested in finding either $Q_\beta$ or if one is interested in quantitative evaluation of flame spread times for different fuels using results presented in Part II.

\[ \text{sensible heat of fuel} \]

\[ h_{r,5} = \text{J fuel/gas} \]