THE EVAPORATION AND COMBUSTION OF A SPHERICAL CLOUD OF DROPLETS*

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ABSTRACT

A recent trend uses a ~ flame around the cloud to model the evaporation, ignition and combustion of spray. Previously a quasi-steady (QS) group ignition analysis has been reported, and numerical results were presented for evaporation and diffusion controlled combustion. Transient conservation equations and explicit solutions for the evaporation and burning rate of the cloud are presented. Transient effects on the results are discussed. The comparison of results between the simple solutions and the numerical method is very good. Based on the explicit solutions, universal plots are given for the nondimensional burning rate of a QS cloud for the problems of pure evaporation and for the diffusion and kinetic controlled combustion. Attempts are made to relate the results of a spherical cloud to study the combustion behavior of conical sprays. It is shown that the combustion of a cloud of radius \( R_c \) consisting of drops of radii \( a \), can be modeled as the combustion of a single drop of radius \( R_c \) for \( C > 30 \) and as the combustion of an individual drop of radius \( a \) for \( C < 0.30 \). A local group combustion criterion is presented. This criterion shows that there may be simultaneous burning of various groups of drops with a single flame around each group within the spray core.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
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<tbody>
<tr>
<td>( M )</td>
<td>mass flow rate</td>
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<td>( \dot{M} )</td>
<td>mass flow rate per unit volume</td>
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<td>( \dot{M}_{\text{c}} )</td>
<td>mass flow rate at the cloud periphery</td>
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<td>( n )</td>
<td>total number of droplets in the cloud</td>
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<td>( n_d )</td>
<td>number of droplets per unit volume</td>
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<td>( D_{\text{Nusselt}} )</td>
<td>Nusselt number based on radius of droplet</td>
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<tr>
<td>( q )</td>
<td>order of reaction with respect to fuel and oxidizer</td>
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<tr>
<td>( q' )</td>
<td>see eq. (19)</td>
</tr>
<tr>
<td>( R )</td>
<td>universal gas constant</td>
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<td>( R_c )</td>
<td>cloud radius (Fig. 1)</td>
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<td>( T_e )</td>
<td>time</td>
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<td>( \alpha )</td>
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<td>( \beta )</td>
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<td>( \gamma )</td>
<td>stoichiometric mass of oxygen</td>
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<td>( \lambda_k )</td>
<td>fuel volume fraction, equation (14c)</td>
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<td>( \lambda_{\text{ref}} )</td>
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<td>( \tau )</td>
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<td>( \tau_{\text{ref}} )</td>
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<td>liquid</td>
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*This paper may be construed as Part II while the earlier paper on Group Ignition (ASME WA/UT-15) can be considered as Part I.
1. INTRODUCTION

In the past, modeling of spray combustion has been treated as a sum of contributions from combustion of individual droplets. Recent studies use a group flame around the cloud to model the evaporation, ignition and combustion of spray as opposed to modeling the behavior with a flame around each drop in the cloud [1-6]. Experimental evidence seems to suggest the combustion as a group behavior [7, 8, 9]. A review on group combustion has been recently carried out in ref. 10. The group combustion behavior will lead to entirely different results for the efficiency, flame stability and pollution (soot, NO, etc.) characteristics, compared to the computational results based on single drop behavior.

The transient analysis of group combustion behavior of a cloud of droplets is complicated by 1) the development of spray, 2) the spray geometry, 3) the presence of aerodynamic flow field surrounding the moving drops, 4) the mutual interactions between the gases and the drops and 5) the interaction between the drops themselves. However, qualitative information could still be obtained by assuming the geometry of a spray as a spherical cloud, and describing the mass loss rate from each drop inside the cloud as pure evaporation rate. Further, it is shown that such treatment leads to simple, but strikingly generalized, results for the group evaporation and combustion rates in a form analogous to the rates for a single droplet. The objectives of the present work are to obtain explicit solutions for the group burning rates and for the flame structure, and to compare the explicit results with the results obtained from the rigorous numerical method.

2. CONSERVATION EQUATIONS

2.1 The Model and Assumptions

More description of the model described here are given by Annapalai et al. [5]. Consider a spherical cloud of radius \( R_c \) containing packed droplets (Fig. 1) in a quiescent atmosphere. The cloud is essentially a two phase zone consisting of liquid and gas phases. The drops in the cloud evaporate. Two zones called film (Zone I) and bulk gas (Zone II) zones (see Fig. 1) are inside the cloud. The zone outside the cloud is a single phase zone (Zone III). The cloud zone is assumed to be a continuum which implies that the droplets act as point sources (e.g., Fig. 1).

The ignition and burning of clouds are assumed to occur as follows: When the cloud of droplets is subjected to a high temperature environment, a thermal wave propagates from the temperature of the cloud. The vapors produced undergo ignition in both Zones I and III. At the run away condition, ignition occurs to occur outside the cloud and ignition temperature of the cloud are estimated to be much lower than compared to the typical values obtained for the radiation from a single droplet [3]. Beyond the ignition condition, the chemical reaction zone extends beyond the cloud. The reaction rate increases. This is rapid enough to effectively prevent the oxygen's reaching the cloud, and to increase outside the cloud. The flame waves lead to the cloud for evaporation of individual drops and for superheating the vapors to the flame temperature. The flame temperature, for QS diffusion controlled combustion is the same as the adiabatic flame temperature [5].

Explicit solutions lead to results in the form of universal plots for the nondimensional burning rate of a spherical cloud. These plots are valid for the problems of pure evaporation and for the diffusion and kinetic controlled combustion. Within the range of group combustion numbers studied, the explicit results for the QS burning rate agree almost exactly with the numerical results.

2.2 Convective Equations

The nonsteady (NS) form of the conservation equations are given below:

1) Mass:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = \frac{\partial h}{\partial t} - \nabla \cdot (h \mathbf{u})
\]

(1)

where \( h_{\text{f},p} = n_{\text{f},p} h_{\text{f},p}' + n_{\text{v},p} h_{\text{v},p}' \)

(2a)

\( h_{\text{f},p}' = n_{\text{f},p} L_{\text{f},p} \ln (1 + \lambda), \quad r < R_c \)

(2b)

\( h_{\text{v},p}' = 0, \quad r > R_c \)

(3)

and \( h_{\text{f},p} \) is computed with a reference condition at the surface temperature of drop.

Equation (1) implicitly assumes that the size of the drop varies only as a function of radius from the center of the cloud and there are a large number of droplets in the cloud (i.e., \( N_{\text{f},p}' \) is a continuous function of \( r \)).

II) Species:

Fuel:

\[ N_{\text{f},p}' \]

Interaction effects can be included by using the results of Lobkowsky [11].
\[ n(t, z) = n_0 \left( \frac{t}{\gamma}, \frac{z}{\xi} \right) = \frac{\gamma}{\xi} \left( \frac{t}{\gamma}, \frac{z}{\xi} \right) = \frac{\gamma}{\xi} \left( n_0, \frac{z}{\xi} \right) = 0 \quad (18b) \]
\[ \zeta \left( \frac{t}{\gamma}, \frac{z}{\xi} \right) = \zeta_0 \left( \frac{t}{\gamma}, \frac{z}{\xi} \right) = 1.0 \quad \gamma \leq \zeta \leq \xi \quad (18c) \]
\[ \alpha(t, z) = 1.0 \quad \gamma \leq \zeta \leq \xi \quad \alpha \leq \zeta \quad (18d) \]

Equations (15), (16), and (17), subject to the initial and boundary conditions (18), can be solved and solution for burning rate can be obtained for a QS problem.

2.5 Quasi-steady solutions and Transient Effects

When a cloud is suddenly subjected to a prescribed ambient temperature, a thermal wave propagates from the ambient to the cloud. The thermal diffusion time scale \( t_D \) is approximately \( R^2 / D_0 \) while the heat up time scale \( t_H \) of a drop of radius \( r_0 \) is approximately \( 4D_0 / r_0 \).

The temperature time scale \( t_T \) for the drop approximately \( 4D_0 / r_0 \). Assuming the cloud radius is 1 cm and the drop radius is 50 μm, they estimate would follow that \( t_D \approx 1 \) s, \( t_H \approx 10 \) ms, and \( t_T \approx 10 \) ms. Since \( t_T \approx t_H \), the thermal wave penetrates partially into the interior of the cloud within a time scale of the order of \( t_T \) or \( t_H \). Thus, the outer layers of the cloud would already have evaporated by the time the thermal wave penetrated into the center of the cloud. Hence, at any given time, only an outer layer of drops participate in the evaporation and combustion processes, while the interior of the cloud may remain cold. The evaporation, ignition and combustion processes of a fuel cloud therefore essentially is a transient process. In order to obtain qualitative information and gain useful insight into the group behavior, more tractable cases (such as the quasi-steady models) must be treated. Surprisingly, such a treatment leads to simple but generalized explicit solutions for the mass loss rate of cloud in a form analogous to the results for a single droplet.

Further, these results will be used in future research to compare the quantitative results for the transient problem, which is the ultimate objective of the present research. Thus, only QS results are presented and the effects of transient processes on the results are discussed. With the QS assumption, the time derivatives will be eliminated from Eqs. (15), (16), and (17), and the steady form of boundary conditions (18b) and (18c) will be used in solving for \( \alpha \) (non-dimensional burning rate), \( n \), \( \zeta \), and \( \alpha \) (flame structure).

3. RESULTS AND DISCUSSION

3.1 QS Integral Solutions and Numerical Results

The integral solutions to the steady form of Eqs. (15), (16), and (17) were previously presented [3]. In principle, the steady form of equations (16) can be solved for \( \zeta \) with two boundary conditions (18b) and (18c). For the diffusion controlled combustion, \( V_0 = 0 \) for \( r > R_c \) and hence \( V_0 \) is known from Eqs. (12c) and (12a). Using Eqs. (12b) and (12a), \( V_0 \) can be solved and hence \( V_0 \) is obtained. Using Eq. (8), \( V_0 \) is then solved. For the pure evaporation problem, \( V_0 \) is known and as such \( V_0 \) is obtained from Eqs. (12b) and (12c). Since \( \zeta < 1 \) for the pure evaporation problem \( \zeta \) profile is immediately obtained using Eq. (13a). Again both \( V_0 \) and \( \zeta \) are unknown from Eqs. (8) and (12a), \( V_0 \) and \( \zeta \) can then be obtained.

In the next, Eq. (21) is obtained for the problem where there are no steady state boundary conditions \( \zeta < 1 \) and \( \zeta \geq 1 \) and the three are needed for the second order differential equation (16) and first order differential equation (17). The results obtained for \( \zeta \), \( \zeta_0 \), and \( \zeta \) are identical and presented in the table below.

1.2 QS Explicit Solutions

Explicit solutions for \( \zeta, \phi \), and \( M \) are given for the steady conservation equations. As \( r \to 0 \), the thermal enthalpy decreases but the fuel mass fraction in Zone II increases. As \( r \to R_c \), the thermal enthalpy increases but the fuel mass fraction in Zone II decreases. Because of the competing effects, the fuel mass fraction at the surface of each droplet \( (V_0, \zeta) \) is expected to remain constant inside the cloud. This statement that \( V_0 \) is constant is true for QS evaporation and diffusion controlled combustion [10]; it is approximately valid for the chemically controlled combustion problem [10]. Thus, \( \gamma \) as given by Eqs. (13c), (13d), and (13f) remain approximately constant inside the cloud. Further, if the drop size distribution is given by the following law,

\[ \zeta = G_1 e^G_1 \quad (19a) \]

then using Eqs. (15) and (16), the following relations can be obtained for \( n \), \( \zeta \), and \( M \) of a chemically reacting cloud (Appendix A of ref. 4):

\[ \alpha = \zeta (q-3)^2 \left( [q(q-2)/2] \left( \frac{2}{q-2} \right) \right) \left( \frac{2}{q-2} \right) \phi = \exp (-M(1 + F (\zeta))) \quad \zeta \leq 1 \quad (20a) \]

\[ \phi = \exp (-M \zeta) \quad \zeta \geq 1 \quad (20b) \]

where

\[ F(\zeta) = 2^{1-q} \left( \frac{2}{q-2} \right) \left( \frac{2}{q-2} \right) \]

\[ 2 \left( \frac{2}{q-2} \right) \left( \frac{2}{q-2} \right) \left( \frac{2}{q-2} \right) \left( \frac{2}{q-2} \right) \left( \frac{2}{q-2} \right) \left( \frac{2}{q-2} \right) \left( \frac{2}{q-2} \right) \]  

The ratio of the burning rate of a cloud of radius \( R_c \) to the burning rate of a single drop of radius \( R_c \) is given as

\[ \frac{(M/M_0)}{R_c} = 1 \left( 1 + \frac{(3-q)}{G_1} \right) \left( 1 \left( \frac{3-q}{2} \right) \left( \frac{2}{q-2} \right) \right) \]

and

\[ M_0 = 1 \left( 1 + B_0 \right) \]

\[ B_0 = \frac{(h_{1,0} + (\gamma_{1,0} - \beta_{1,0} \gamma_{1,0} h_{1,0}))}{(1 + (\gamma_{1,0} - \beta_{1,0} h_{1,0}))} \quad (22a) \]

It should be noted that the relation for \( M_0 \) given in Equation (21) is true whether there is pure evaporation, chemical reaction or diffusion controlled combustion of a spherical cloud.
As expected, the agreement is excellent. Equation (21) illustrates that the ratio \( \frac{M}{M_s} \) is simply a function of \( G_1 \) and \( \alpha \). The results for \( M/M_s \) vs \( G_1 \) with \( \alpha = 1 \) as parameter are plotted in Figures 2a and 2b. The value of \( G_1 \) is the same as G-number, with \( \alpha = 0 \) (monodisperse droplet clouds with uniform fuel volume fraction). For the case \( \alpha = 0 \) (case A) as \( G_1 \) is reduced (decreasing \( a_0 \), constant \( n \), the mass loss rate decreases. For \( \alpha = -1 \) (case B) the burning rate of a \( G_1 \) cloud at given \( G_1 \) is lower compared to case A. For a cloud with uniform drop distribution, the parameter \( \alpha = -1 \) implies that the radius of the drop increases linearly with radius from the center of the cloud (pressure jet atomizers). If the \( G_1 \) number of case (B) has to be the same as that of case (A), then the average radius of droplets inside the cloud for case (B) is estimated to be smaller than the radius of drop for case (A). Thus the mass loss rate is lower for the case (B).

As the cloud becomes denser (increasing \( G_1 \)) more and more drops are introduced for denser sprays. More surface area is now available for the evaporation and the evaporation rate increases. As the G-number increases, the densely populated drops near the outer layer cool the gas rapidly to the boiling point temperature and hence the interior drops do not participate in the evaporation process. Thus at high G-numbers the cloud behaves like a single drop of radius \( R_c \) but with the same density as the cloud.

Hence, the burning rate of a cloud of radius \( R_c \) approaches the burning rate of a single drop of radius \( R_c \) at higher values of \( G_1 \). At this limit the bulk gas temperature (Zone III) approaches the boiling point temperature of the fuel.

The G-number could be interpreted as a ratio of heat/mass transfer between the bulk gas zone (Zone III) and the drop to the maximum possible heat/mass transfer between the cloud surface and the atmosphere. This was shown in Figure 2b with the corresponding burning rate of \( M/M_s \) at \( \alpha = 0 \). This relationship is based on the assumption that the burning rate of a single drop of radius \( R_c \) is not affected by the presence of other drops.

$$G_1 = \frac{C_1}{C_{1,5}}$$

Thus for \( G_1 \) in the range of \( 3-10 \), a ten-fold reduction in G-number takes place. As observed from Figure 2a, the effect on the burning rate is significant between \( \alpha = 1 \) and 10. The effect is not significant at high G-numbers (heat transfer limited in the single droplet zone and low G-number heat transfer limited in the two drop cloud zone).

In order to determine the effects of the G-number on the burning rate, the burning rate of a cloud of droplets (M) should be compared to the total burning rate (M.), which could be obtained if each droplet in the cloud burns as though it is located in an infinite atmosphere. A derivation is presented in Appendix A to obtain the ratio of M/M., as a function of G1 and \( \alpha = 0 \).
The dimensionless burning rate vs modified group combustion number $N_f$, total burning rate of all drops, in the event each drop burns individually.

Fusion controlled combustion. As expected, $N_f/M = 1.0$ as $G_i = 0$ (Gillut clouds), since droplets are located so far apart that each drop appears to evaporate in an infinite atmosphere.

For diffusion controlled combustion, setting $Y_f = \frac{Y_f}{N_f}$ in Equation (12e) and using Eqs. (12a) and (20b), the flame location is determined.

$$\frac{R}{R_f} = \frac{1}{N} \ln \left(1 + \frac{Y_f}{N_f} \right) \quad (24a)$$

Rewriting eq. (24a) and using eq. (23b),

$$\ln(1 + \frac{R}{R_f}) = \left(\frac{\ln(1 + \frac{Y_f}{N_f})}{N_f} \right) \times \frac{N/N_f}{N_f} \quad (24b)$$

As $G_i$ decreases, the blowing rate, $N_f$ decreases (Figs. 2) and consequently the flame moves towards the cloud. At the critical point, $N_f$ is such that $R/R_f = 1.0$, which implies that the flame is located just at the radius of the cloud. Thus at the critical condition,

$$\frac{N_f}{N} = \frac{1}{N_f} \ln \left(1 + \frac{Y_f}{N_f} \right) \quad (24c)$$

From Equation (24c) and using Figs. 2a, 2b and 3, the corresponding $G_i$ can be determined. For $G_i < G_i(c)$, diffusion controlled burning rates from Figs. 2 and 3 should not be calculated (see Section 3.6). Another point to be noted is that there is no solution for the non-dimensional burning rate $R/R_f > 2$. The mass of the cloud tends to reach infinity as $r = 0$ for $q = 2$, and is therefore not a realistic case. Figs. 2 and 3 are also valid for combustion under high pressures such as those existing in diesel engines. However caution must be exercised in estimating 1) the fuel volume fraction (higher at high pressure), 2) the $G_i$-number and 3) the transfer number $N_f$.

3.4 Criteria for Modeling Spray Combustion

Figs. 2 and 3 can be used to classify the sprays and to describe the qualitative combustion behavior of a spray. Chiu and Liu [4] defined high $G_i$ spray as the spray with the $G_i$-number significantly higher than unity and low $G_i$ spray as a spray with $G_i$-number lower than unity. Since $G_i = \rho G_i$ for a uniform cloud with uniform drop distribution, Figs. 2a and 3 offer an alternative method of classifying the sprays. From Fig. 2a, observations could be made that the cloud mass loss rate increases rapidly between $G_i = 0.3$ and 30. Beyond $G_i = 100$, the cloud mass loss rate is almost 20.

of the mass loss rate of a single drop, with the same radius as that of the cloud. Below $G_i = 0.3$, the cloud mass loss rate is approximately 10% of the single drop. By drawing the tangent lines AB and CD as shown in Figs. 2a and 3, the low $G_i$ spray can be defined as the spray with $G_i < 0.3$ (Point A, Fig. 2b), and the high $G_i$ spray can be defined as the spray with $G_i > 40$ (Point B, Fig. 2b). It is apparent from Fig. 3 that the low $G_i$ sprays can be modeled as the spray with individual drop combustion. However, it obvious that the high $G_i$ spray can be modeled as group/chain combustion with mass loss rate almost the same as the rate of a single drop with the radius equal to the cloud radius and with the density of drop being equal to the cloud mass per unit volume. Note that spray, ambient or fuel properties have not been used in classifying the sprays.

3.5 Relation between $G_i$-number of Spherical Cloud and Fuel Sprays

![Diagram](image.png)

**Fig. 4.** (a) Spray combustion and modelling
(b) Group combustion number vs distance from injector
(c) Burning rate vs distance from the injector

The $G_i$-numbers defined for spherical clouds can be approximately extended to the spray cones by using the following procedure. Consider a conical spray of half angle $\theta$ as shown in Fig. 4. As the spray develops, the droplets are formed and they travel with a velocity of $V_d$. If it is assumed that the air velocity is the same as the drop velocity, then evaporation or combustion takes place under quiescent conditions. Over a time interval of $t'$, the spray penetrates a distance $\theta t'$. If there is no significant evaporation within this time $t'$, the mass conservation yields

$$m(t') = m_0 - \frac{1}{2} \pi r^2 \rho \frac{V_d}{\rho} \theta t'$$

$$m(t') = m_0 - \frac{1}{2} \pi r^2 \rho \frac{V_d}{\rho} \theta t'$$

(25)
\[ n_{\infty} \rightarrow 0 \quad \text{as} \quad r \rightarrow 0 \quad (34) \]

which indicates that the temperature of the bulk phase approaches the temperature of the liquid drop.

3.8 Transient Effects under Combustion Conditions

From Fig. 8, it is clear that significant mass is vaporized only near the outer layer of the cloud for \( G > 10^3 \). Because the bulk phase in the interior of the cloud is near the boiling point of the liquid drops (of Equation 34), a transient model under high \( G \)-number is expected to yield somewhat similar results since the interior is near the initial temperature of the drops without significant evaporation while the droplets in the outer layer are vaporizing. However at low \( G \)-numbers, according to QS model, significant evaporation occurs deep inside the cloud while a NS model suggests that the evaporation occurs only in the outer layers of the cloud. Thus the NS model at first is expected to yield reduced burning rates compared to the results for a QS case. Since vaporization is reduced, the flame position will be closer to the cloud periphery which increases the bulk temperature for the drops in the outer layer which tend to increase the vaporization rates. Thus QS cloud burning rate may not be significantly different from the NS cloud burning rate. Still it remains to be proven with results from the NS model.

4. SUMMARY

1. A transient formulation of group combustion has been developed. With suitable Schub-Zelbovich variables the conservation equations are simplified.
2. Explicit solutions are given for the evaporation and combustion of a QS cloud.
3. Universal plots are given for the nondimensional burning rate. These plots reveal that a cloud of radius \( R_c \) consisting of drops of radius \( a_0 \) can be modeled as combustion of a single drop of radius \( R_0 \), if \( G > 30 \) and as the combustion of individual droplets of radii \( a_0 \), if \( G < 0.30 \).
4. A criterion is also presented for local group combustion.
5. Universal plots are given for the flame structure of the cloud. These results are useful in determining the fuel and oxygen mass fraction and temperature profiles for both the evaporation and combustion problems.

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6. REFERENCES


Appendix A. Relation between group burning rate and total burning rates of single drops

The burning rate of a single drop is given as

\[ \dot{m}_D = 4\pi D a_0 \ln (1 + b_s) \quad (A-1) \]

If it is assumed that every drop inside the cloud burns as though it is located in an infinite atmosphere, total burning rate is given as

\[ \dot{m}_t = (4\pi)^2 \rho_b \ln (1 + b_s) \int_0^{R_c} n a_0^2 \tau d\tau \quad (A-2) \]

Using eqs. (14b) and (14c) to eliminate \( a_o \) and \( n \) from eq. (A-2) and using eqs. (14c), (14d) and (14a),

\[ \dot{m}_t = 4\pi D R_c \ln (1 + b_s) \int_0^{1.0} \frac{G_c}{\zeta^3} d\zeta \quad (A-3) \]

Using eq. (18d) in (A-3) and integrating.

\[ \dot{m}_t = 4\pi D R_c \ln (1 + b_s)/(3-q) \quad (A-4) \]

Using eqs. (14j) and (21) in eq. (A-5), the eq. (23) is obtained.
Fig. 6. Coupling function and local mass loss rate vs nondimensional distance. Comparison between the explicit and numerical results.
Fuel: dodecane

Fig. 7. Generalized plot for coupling function profiles at various $G_1$-numbers and $q$ values

Evaporation problems.
It should be observed from Fig. 7 that

\[ -\ln \frac{\phi}{M} \text{ as } q \to 0, \quad (r > 0) \]  

\[ \text{It can be shown from Eq. (28a) that } c = \text{ (30) } \]

Using Eqs. (12a) and (12c) for $\Gamma$ and Equation (22a).

\[ \Gamma + 1 = \frac{K}{1 + s_H} x/\gamma (M/M_0) \]  

For large $G_1$-numbers, steam combustion occurs ($M/M_0 > 1.0$, Fig. 7) and $\Gamma \to 1.0$. Thus from Equation (31),

\[ \text{It can be shown from Eq. (28) that } \]

\[ \text{Fig. 8. Local mass loss rate profiles at various } G_1\text{-numbers}\]

(a) $q = 0$  (b) $q = -1$  (c) $q = +1$

\[ Y_{F.r} \to 0 = \frac{1}{1 + \gamma s_1/(1 + s_2)} \]  

which is the fuel mass fraction in the bulk phase. This result is exactly same as the surface fuel mass fraction of a single droplet [12]. Using equation (32) in Eqs. (12a) and (12b), the thermal enthalpy of the bulk phase is determined as $r \to 0$.

\[ \frac{\ln 1}{1} = \left(1 + s_2\right)^{1-1/\gamma} \]  

\[ \left(1 + s_1\right)^{1-1/\gamma} \]  

\[ \left(1 + s_2\right)^{1-1/\gamma} \]