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OPTIMIZATION OF AIR COOLED CONDENSER

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ABSTRACT

This paper presents essential design parameters for determining the optimum configuration for an air cooled condenser. The procedure is illustrated for a plant operating on a Rankine cycle with air as the coolant. The inclusion of the engine power necessary to drive the condenser fan yields the optimum values for the air velocity and condenser temperature. These optimum values depend on the parameter to be optimized and the parameters of interest differ depending upon the type of application. The results of this study allow the selection of condenser temperature and air velocity which will produce either, (i) minimum frontal areas (ii) minimum heat transfer area for a fixed condenser volume or (iii) maximum thermal efficiency based on useful work output. The analyses for cases (i) and (ii) are carried out with and without the assumption the air exit temperature is equal to the condenser temperature so that the effect of this assumption used in previous work may be determined. Results for a Rankine cycle using water with 68 bars boiler pressure show that the condenser temperature and air velocity producing the minimum frontal area is about 135°C and a high 94 m/s respectively. The net thermal efficiency at this condition is 18% and the condenser frontal area 4 cm²/kw.

For the same water Rankine cycle with 68 bars boiler pressure optimisation of the net thermal efficiency with a fixed condenser and air velocity yields a condenser temperature of 85°C and a net thermal efficiency of 21.5%. All results show the desirability of high air velocities even with accompanying high air power requirements. All results are given in nondimensional form so that they are more generally applicable.

NOMENCLATURE

\( A \) area (mean area if not subscripted)
\( a \) a constant dependent upon type of low [see equation (8)]
\( B \) equation (28a); also Table II
\( b \) fin width (Fig. 1)
\( C_f \) friction coefficient
\( C_p \) specific heat at constant pressure of the air
\( D_R \) height of the fin (Fig. 1)
\( h \) heat transfer coefficient
\( m \) an exponent in Reynolds's number
\( \alpha \) \( \frac{1}{2} \) for turbulent flow
\( \beta \) \( \frac{1}{2} \) for laminar flow
\( \eta \) number of fins per unit length
\( P \) pressure
\( Pr \) Prandtl number
\( \dot{Q} \) heat rate
\( R \) gas constant
\( Re \) Reynolds's number
\( T \) temperature
\( V \) velocity
\( V_C \) volume occupied by the condenser
\( \dot{W} \) work per unit time
\( x \) distance along the flow stream (Fig. 1)
\( \alpha \) a function dependent upon \( T_C \) and \( T_a \) only equations (14a) and (15a)
\( \beta \) ratio of fan work to useful work
\( \gamma \) ratio of specific heats at constant pressure and at constant volume
\( \eta \) efficiency
\( \nu \) kinematic viscosity of the cooling medium
\( \rho \) density of the cooling medium
\( \tau \) shear stress
\( \phi \) equations (44) and (52b)

Subscript
\( I \) input to power plant
\( a \) ambient
\( act \) actual
\( b \) boiler condition
\( c \) condenser
\( e \) exit condition of the air
\( f \) fin, fan
\( i \) inlet, frontal
\( \text{min} \) minimum
\( o \) outlet, frontal
\( \text{opt} \) optimum
\( \text{useful} \)
\( w \) wall
1. INTRODUCTION

The condenser must remove and recover essentially all the waste heat produced by the Rankine cycle power plant. There exists a considerable interest in the development of automotive Rankine cycle power plants because of their adaptability to different fuels, continuous combustion and relatively less pollutants in the exhaust [1,2,3,4]. Since the water circulation is about ten times more than the fuel, a condenser is an essential feature for an automotive Rankine powered vehicle in order to form a closed cycle rather an open cycle. The condenser in an automotive type application is a large and expensive component of the power plant. A finned condenser design is proposed. The power, requiring to perform the cooling task is thereby considerable, requiring a larger power plant to drive the fan and producing a lower net engine efficiency. Also the volume and frontal area of the condenser become dominating factors in determining the vehicle configuration and the cost. Optimization of the condenser system is then especially important in the design of an automotive Rankine cycle power plant. Even though the procedure is developed with automotive type application, a similar analysis can be carried out for the design of condenser depending upon the industrial needs.

The condenser design variables, i.e. frontal area, depth, air velocity, and fin spacing may be optimized with respect to several different parameters depending on the application. The optimized variables might be thermal efficiency of the engine, condenser frontal area (automotive applications) condenser heat transfer area, or condenser volume (marine applications). In actuality all these latter parameters are usually of some importance and a trade-off between the variables is necessary. The designer needs to know what the optimum design is for each parameter, however, so that a trade-off can be properly made between these parameters.

Stuart, Duettio, and Zoucha (5) recently carried out optimization calculations of such condensers by finding the condenser temperature which produces the minimum frontal area. They held constant the total engine output and the air velocity and carried out the analysis for a Carnot Engine and a Rankine cycle using mercury, water, and R-113 as working fluids. They also looked at a second case of finding the air velocity through the condenser which would minimize the total condenser heat transfer area for a fixed condenser temperature and useful power output, i.e. the total power less the condenser fan power). In all their work, Stuart, et al., set the exit air temperature from the condenser equal to the condenser temperature.

A more general, comprehensive analysis is useful to determine the optimum value of both the air velocity and condenser frontal area which produces the optimum frontal area, heat transfer area or the thermal efficiency.

The condenser designer is faced with making an optimum choice of condenser frontal area A', depth X', velocity U', and fins per inch Np. For given heat rejection rate Qr, it is then fixed the condenser temperature Tc and the fan power Wf which in turn fixes the useful power Wn and the thermal efficiency \( \eta = \frac{Q}{W_n} \) based on useful net power output. It is worthwhile to note the qualitative effect of the design variables, frontal area, depth and fins per meter.

As the frontal area, number of fins, or depth is increased with constant air velocity through the coil and useful power output, the condenser temperature will drop, increasing the thermal efficiency based on the total engine output. However, the fan work increases which means for constant usable power output, \( W_f \), the engine total power output must increase which tends to decrease \( \eta = \frac{Q}{W_n} \) and increases \( \Gamma \). With these opposite effects there is some value of these parameters A', Np and which will produce either the maximum useful efficiency, minimum frontal area, or minimum heat transfer area for a fixed condenser volume.

On the other hand, as the air velocity is increased for fixed A', X', Np, and Wf, the fan power goes up requiring increased total engine power but the engine total efficiency goes up due to decreasing \( \Gamma \). For any given condition some value of air velocity will produce either the maximum useful thermal efficiency, a minimum condenser frontal area, or minimum heat transfer area for a fixed condenser volume. The complex interactions which must be considered when trying to select these condenser design variables require a comprehensive analysis.

This work carries out such an analysis first developing the equation and then non-dimensionalizing them to make the results more generally applicable. Several cases are then studied as shown in Table I along with the varying assumptions. The results of the study allow the selection of condenser temperature and air velocity which will produce i) minimum frontal area, ii) minimum total heat transfer area for a fixed condenser volume, or iii) maximum thermal efficiency based on useful work output. The analyses for cases (i) and (iii) are carried out with and without the assumption that the air exit temperature is equal to the condenser temperature so that the effect of this assumption may be determined. All results are given in non-dimensional form so that the results are more generally applicable.

2. GOVERNING EQUATIONS

2.1. Assumptions

The assumptions taken throughout the analysis are as follows:

(i) Only wall frictional work is considered in moving the cooling medium through the condenser. (ii) The fins around the condenser form rectangular air flow passages (Fig. 1). (iii) The friction coefficient is related to the Reynolds number by a general expression, \( \frac{C_k}{e} = a(R)e^{-b}. \) (iv) The velocity of air through the condenser is constant. (v) The condenser heat transfer surfaces in contact with the air are at the Rankine cycle working fluid temperature. (vi) The Reynolds analogy is applicable for the relation between heat transfer and frictional coefficients.

2.2. The Equations

The efficiency of a heat engine is given as

\[ \eta = \frac{W_n}{Q} \]  

where

\[ W_n = U' \eta + W_f \]  

From an energy balance for the cooling air and the condenser and using assumption (iv),

\[ \dot{Q}_c = \rho A' \left( U' \dot{W} - C' \right) \frac{U'}{T_0} \]  

(1)

(2)

(3)
Using the law of conservation of mass,
\[ \rho \frac{A}{\bar{v}} V' = \rho \frac{A_0}{V_0} V' \]  
(4)
The heat thus rejected by the fluid in the condenser must be transferred to the coolant air by combined forced and natural convection. Assuming that \( T_0 = T_0' \) and from Newton's law of convection, taking the average air temperature to be halfway between its entrance and exit temperatures
\[ \bar{v}_c = \frac{\Delta T}{T_0'} \frac{T_0' - T_0}{T_0} \]  
(5a)
where \( A_0' \) can be obtained as follows for the given configuration in Fig. 1:
\[ A_0' = 2 \frac{n}{L} A_x' \]  
(5b)
Using assumptions (1), (11), and (111),
\[ \frac{\Delta T}{T_0'} = \frac{\text{Pr} 2/3}{\sqrt{\text{Pr} \text{Ct}'} V^* C_0^*} = C_f / 2 \]  
(6)
where \( T^* = C_f \rho V^* \sqrt{\text{Pr} / \text{Ct}'} \),
\[ C_f = \frac{a/(Re C_t')}{10} \]  
(7)
With assumption (vi),
\[ \bar{v}_c = \frac{\Delta T}{T_0'} \frac{T_0' - T_0}{T_0} = \frac{n}{L} A_x' \]  
(8)
\[ \text{Pr} 2/3 \sqrt{\text{Pr} \text{Ct}'} V^* C_0^* = C_f / 2 \]  
(9)
Note that the constants \( m \) and \( a \) in equation (6) vary depending upon the flow regime. For the case \( T_0' = T_0 \), there are only two independent parameters \( V^* \) and \( T_0' \).

2.3. Transformations

In this section, normalizations for the dimensional quantities are carried out. Let \( V' = V^*/V^* \), where \( V^* = \frac{C_t'}{T_0'} / \text{Pr}^{2/3} \); 
\[ A_0' = A_x'/A_x \], where \( A_x' = (\rho^2/\text{Pr})(\alpha - 1)/\alpha; T^* = T_0' / T_0; \beta = \frac{W_0'}{W_0} \]  
(10a,10b,11a,11b,12a,13b)

3. Solutions

The present section carries out optimization of design parameters when \( T_0' = T_0 \). Section 3.1 presents relevant equations for the frontal areas. As well as, frontal areas are minimized with respect to velocity of air and the condenser temperature holding useful constant. Section 3.1.1 presents optimization results for frontal areas for a condenser operating on any general cycle while section 3.1.2 presents optimization results when such general cycle efficiency could be replaced by a straight line fit, \( \eta = B - D T_0 \). Section 3.2 presents results when heat transfer area is minimized based on constant useful power output and specified number of fins per unit length. Section 3.3 carries out minimization of heat transfer area when useful power output and space volume occupied by condenser are specified. Section 3.4 presents solutions when net efficiency based on useful power output is maximized.

Using equations (10) to (13),
\[ \eta = \frac{1 - n}{n(T_0' - 1)} \]  
(14)
where, \( \eta = A_0 V/(1 + B) \)
Similarly, \( \alpha = \frac{(1 - n) T_0}{n(T_0' - 1)} \)
where, \( \alpha = A_0 V/(1 + B) \)
If the curve for \( \eta \) vs. \( T_0 \) is known for a given heat engine, equations (14) and (15) can be plotted against \( T_0 \) for given ambient conditions. Looking at the extreme cases for \( T_0 \), it is readily seen that as \( T_0 \to 1 \) or \( T_0 \to T_0' \), \( \eta \to 0 \). Since the nondimensional condenser temperatures \( T_0 \) has to satisfy the inequality \( 1 < T_0 \to T_0' \), then \( \alpha_1 \) or \( \alpha_0 \) has to pass through a minimum when \( T_0 \) is varied from 1 to \( T_0' \).

Fig. 2 shows a plot of \( \eta \) vs \( T_0' \) for various boiler pressures operating on Rankine cycle, including a plot to convert the scale from \( T_0' \) to \( T_0 \) for various ambient conditions. Figs 3a and 3b show the variation of \( \alpha_1 \), \( \alpha_0 \), and \( \alpha_0 (\alpha_1)^{1/2} \) with respect to \( T_0' \) with \( \eta \) as a parameter.

3.1. Frontal Areas

Since the frontal area varies with respect to the condenser temperature and air velocity, the temperature and air velocity which produces the minimum frontal area are determined as follows.
\[ \beta = \frac{2V_0^2 \alpha}{(1 - 2V_0^2 \alpha)} \]  
(16)
\[ \left[ \frac{2}{V_0} \alpha_0 (2 \alpha_0)^{1/2} \right] = \frac{1}{V_0} \left[ \alpha_0 (2 \alpha_0)^{1/2} (1 - 2V_0^2 \alpha) \right] \]  
(17)
It is seen that for positive values of \( \alpha_1, \alpha_0, (2 \alpha_0)^{1/2} \), the velocity group \( V_0^2 \alpha_0 \) must lie between 0 and \( 1 \). Also, it is seen that \( \alpha_1 = \alpha_0^2 \) at both limits. Similarly,
\[ \left[ \frac{\alpha_0}{2V_0} \alpha_0 (2 \alpha_0)^{1/2} \right] = \frac{1}{V_0} \left[ (2 \alpha_0)^{1/2} (1 - 2V_0^2 \alpha) \right] \]  
(18)
Fig. 4a illustrates the variation of \( \alpha_1, \alpha_0, (2 \alpha_0)^{1/2} \) with respect to \( V \). (2 \alpha_0)^{1/2} \), the minimum value for the inlet area of condenser can be shown to occur at
\[ V_0 \left( \frac{2 \alpha_0}{\sqrt{2}} \right)^{1/2} = 1/3 \]  
(19)
Substituting (19) in (16), \( B = 1/2 \). Note that this shows the minimum condenser frontal area occurs when the condenser fan power is one-half the useful engine output. Substituting equation (19) in (17) and (18), optimum areas with respect to the velocity are given as,
\[ \frac{A_0}{A_0} \left( \frac{2 \alpha_0}{\sqrt{2}} \right)^{1/2} = \frac{A_0}{A_0} \left( \frac{2 \alpha_0}{\sqrt{2}} \right)^{1/2} = 3/2 \]  
(20)
Since \( \alpha_1 \) and \( \alpha_0 \) are functions of \( T_0 \), the minimum inlet and outlet areas occur at the conditions when \( \alpha_1 \) and \( \alpha_0, \alpha_0 (2 \alpha_0)^{1/2} \) are minimum. In order to evaluate the absolute optimum areas quantitatively, \( \alpha_0 (2 \alpha_0)^{1/2} \) must be evaluated. Two methods are examined. (i) exact method and (ii) approximate method.

3.1.1 Exact Method: The quantities \( \alpha_1 \) and \( \alpha_0 \) are defined by equations (14) and (15). The efficiency \( \eta \) is to be determined for a given cycle. For example, for a Rankine cycle powered vehicle, figures 2 and 3 are to be combined to determine \( \alpha_1 \) and \( \alpha_0 (2 \alpha_0)^{1/2} \). Fig. 7 shows such a plot. For more details refer to the numerical example in Section 5. The optimum \( \alpha_0 \) is seen to occur at \( T_0 \) of 1.35 and optimum \( \alpha_0 \) is seen to be at \( T_0 \) of 1.26.

3.1.2 Approximate Method: Fig. 2 shows that the efficiency vs. \( T_0 \) is almost linear for a Rankine cycle. Hence for any cycle
\[ \eta = \frac{B - D T_0}{T_0} \]  
(21a)
where \( D = D_0/T_B \)  
(21b)
Table II tabulates the constants B, D and D₀ for a Rankine cycle. Substituting equation (24) in (14a) and (15a)

\[
\theta_\text{c}^\text{min} = \frac{(1-B+D_{\text{c}})/(B-D_{\text{c}})(T_{\text{c}}-1)}{D}
\]

Differentiating equation (22) with respect to \(T_{\text{c}}\), and equating the differential to zero for minimum \(\theta_\text{c}^\text{min}\), one can obtain

\[
T_{\text{c}} = \left[ B - 1 + (D + 1 - B \theta_\text{c}^\text{min})^2 \right]/D
\]

Using equation (24) in (22),

\[
(\theta_\text{c}^\text{min})^2 = D/[1 - (1 - B + D)]^2
\]

Similarly for minimum \(\theta_\text{o}^\text{min}\),

\[
T_{\text{c}} = \left[ B + (1 - B + D \theta_\text{o}^\text{min})^2 \right]/(B/D + D)
\]

and

\[
(\theta_\text{o}^\text{min})^2 = D[(B + (1 - B + D \theta_\text{o}^\text{min})^2)/B/D]^2
\]

Though this is an approximate method, the deviation between this method and the absolute method does not appear to be more than 1.5% for \(T_{\text{c}}^\text{opt}\) and about 5% for \(\alpha_1^\text{min}\).

### 3.2 Heat Transfer Area

The heat transfer area, to a large extent, determines the condenser cost and also indicates the space necessary for accommodation. Fig geometry varies as shown in Fig. 1 where the outlet area is larger than the inlet area in order to maintain a constant velocity. Then,

\[
A_\text{w}^\text{min} = 2 n f \frac{h'}{h} b' x' \frac{v'}{v}
\]

where, \(h'/h = A_\text{w}^\text{min}\)

If space occupied by the condenser per unit amount of the useful power is given as \(v_{\text{c}}\)

\[
\frac{A_\text{w}^\text{min}}{v_{\text{c}}} = 2 n f \frac{h'}{h} b' x' \frac{v'}{v}
\]

where metal thickness is assumed to be negligibly small. Note that for given \(n f\), minimum volume is obtained when there is minimum heat transfer area. Making use of equations (28), (29), (18), (19), (17) and (8),

\[
A_\text{w} = 1/\{(V(2a_1 \theta_\text{c}^\text{min})(1-2m)/1-m \left[1-2V^2 a_1 \theta_\text{c}^\text{min}\right]\}
\]

where

\[
\nu = \frac{A_\text{w}}{(2a_1 \theta_\text{c}^\text{min})^{3/2-m}} \frac{T^{2/3}}{P_a^{2/3}} \frac{2 \text{Pr}}{\text{Pr}^{2/3}} \frac{V}{2na_1 \theta_\text{c}^\text{min} \nu^2/m^2}
\]

Obviously, \(A_\text{w}\) has a minimum with respect to \(V\) within the limits \(0 < V < (a_1 \theta_\text{c}^\text{min})^2 < 1/2\) (see eq. 31).

Fig. 4b shows the plot of \(A_\text{w}\) with \((a_1 \theta_\text{c}^\text{min})^2\) for a small range of values of \(m\). Differentiating equation (22) with respect to \(V\) \((a_1 \theta_\text{c}^\text{min})^2\), the velocity \(V\) cor-responding to minimum heat transfer area is given by,

\[
V^*(2a_1 \theta_\text{c}^\text{min})^{3/2} \theta_\text{c}^\text{opt} = [(1/2 - m)/(3/2 - 2m)]^{1/2}
\]

Substituting equation (33) into equation (31),

\[
\frac{A_\text{w}^\text{min}}{v_{\text{c}}} = \left[2 \frac{m - 1}{2 (1 - m)} \left(\frac{2m - 3}{3 - 2m}\right)^{1/2} \right] \frac{T^{2/3}}{P_a^{2/3}} \frac{2 \text{Pr}}{\text{Pr}^{2/3}} \frac{V}{2na_1 \theta_\text{c}^\text{opt}}
\]

Note that \(\theta_\text{c}^\text{opt}\) is a function of \(\alpha_0\) and \(\alpha_1\) which are functions of \(T_{\text{c}}\). Using equations (34) and (32), the minimum \(A_\text{w}^\text{min}\) is given as:

\[
\frac{A_\text{w}^\text{min}}{v_{\text{c}}} = \left[2 \frac{m - 1}{2 (1 - m)} \left(\frac{2m - 3}{3 - 2m}\right)^{1/2} \right] \frac{T^{2/3}}{P_a^{2/3}} \frac{2 \text{Pr}}{\text{Pr}^{2/3}} \frac{V}{2na_1 \theta_\text{c}^\text{opt}}
\]

The exponent \(m\) in the expression for the frictional coefficient is of the order of 1/3 for turbulent flows, while for laminar flows, the value is 0.5. Under turbulent conditions, one looks for \(([(2a_1 \theta_\text{c}^\text{min})^{1/3}/a_1 \theta_\text{c}^\text{min}]^2\) min. It is obvious that \(a_1\) is the dominating term compared to \(a_0\). Moreover, the values of \(T_{\text{c}}\) at which minima of \(\alpha_0\) and \(\alpha_1\) occur are very close to each other. Hence, the temperature \(T_{\text{c}}\) at which the minimum \([(2a_1 \theta_\text{c}^\text{min})^{1/3}/a_1 \theta_\text{c}^\text{min}]^2\) occurs, is taken to be the same as the \(T_{\text{c}}\) at which the minimum occurs. Thus at least for turbulent flows

\[
\frac{A_\text{w}^\text{min}}{v_{\text{c}}} = \left[((11/8)(11/3)^{3/8})[V^2/2na_1 \theta_\text{c}^\text{min}]^{1/4}(2)^{13/8} / a_1 \theta_\text{c}^\text{opt}\right]^{1/8}
\]

Fan power at this optimum condition is given for

\[
\beta = (1/2 - m)/(1 - m)
\]

For turbulent flows \(m = 1/5\) and hence \(\beta = 3/8\)

### 3.3 Number of Fins per Unit Length When Condenser Volume is Specified

As observed from equation (30), the heat transfer area is directly proportional to \(n_f\). For given \(v_{\text{c}}\), minimum area means the minimum number of fins per unit length. Eliminating \(n_f\) in terms of \(v_{\text{c}}\) and \(\alpha_1^\text{min}\) in equations (31) and (32),

\[
A_\text{w} = 1/\{(V(2a_1 \theta_\text{c}^\text{min})(1-2m)/1-m \left[1-2V^2 a_1 \theta_\text{c}^\text{min}\right]\}
\]

where

\[
\rho \frac{V w^2 v_{\text{c}}}{c} = \frac{\alpha_0}{(3/2 - 2m)^{1/2}} \frac{2 \text{Pr}}{\text{Pr}^{2/3}} \frac{V}{2na_1 \theta_\text{c}^\text{min} \nu^2/m^2}
\]

\[
\theta_\text{c}^\text{opt} = (1/2 - m)/(3/2 - 2m)^{1/2}
\]

\[
\frac{2 a_1 \theta_\text{c}^\text{min}}{\alpha_1^\text{opt}} = \frac{2 \text{Pr}}{\text{Pr}^{2/3}} \frac{V}{2na_1 \theta_\text{c}^\text{min} \nu^2/m^2}
\]

\[
\frac{2 a_1 \theta_\text{c}^\text{min}}{\alpha_1^\text{opt}} = \frac{2 \text{Pr}}{\text{Pr}^{2/3}} \frac{V}{2na_1 \theta_\text{c}^\text{min} \nu^2/m^2}
\]
equations (38) and (31) are identical except for the
definition of $A'$. Hence optimum velocity is as given
by equation (33). Also $\beta$ is unchanged. Thus optimiz-
ing with respect to the velocity term,
$$
\left( \frac{A'_w}{(2a_1)^{3/2}} \right) \frac{(2/3) \pi pr^{2/3}}{\gamma'} y^2 = \frac{\gamma' \omega y}{\gamma'}
$$

$$
\frac{\gamma}{1-m} \left[ \frac{(3/2 - 2m)(3 - 4m)/2}{(1/2 - m)(1 - 2m)/2} \right] \frac{1}{1-m}
$$

Fig. 5 plots the left hand side of equation (40) vs.
(\$P'^2 y^2/(2a_1 a_1 y')$) $(\gamma' \gamma - 1)$ with $m$ as a parameter.
Proceeding with similar arguments as given in Section
3.2, the minimum \[ (2a_1)^{3/2} = m/\omega \] occurs at the same
temperature $T_c$ as that one for $(a_1)_{\text{min}}$. Thus the fac-
tor $[(2a_1)^{3/2} - m/\omega]_{\text{min}}$ is evaluated. Corresponding
fin selection is then obtained from equation (3a).

$$
\eta_\text{f} = \frac{1}{\gamma'} \left( \frac{A'_w}{(2a_1)^{3/2}} \right) \frac{(2/3) \pi pr^{2/3}}{\gamma'} y^2
$$

$$
\eta_\text{net} = \frac{(1 - \gamma' \alpha_1)}{(1 - \gamma' \alpha_1)}
$$

3.4 Net Efficiency

From the point of fuel economy, one has to maxim-
ize the net efficiency.

$$
\eta_\text{net} = \frac{\gamma'}{1 + \beta} = \frac{\gamma'}{\gamma'}
$$

Using equation (19) in (42),

$$
\eta_\text{net} = \eta (1 - \gamma' \alpha_1)
$$

where

$$
\gamma' = 2 \gamma^2
$$

Since $\gamma'$ is a function of $\gamma$, while $\alpha_1$ and $\eta$ are functions of $T_c$, one finds that $\eta_{\text{net}}$ is maximum with respect to $\gamma$, if $\gamma = 0$. This means that one require
infinite large frontal and heat transfer area to have a
finite heat transfer rate. Hence, with equation
(21a) and using equation (22), $\eta_{\text{net}}$ is obtained as a
function of $\gamma$ and $T_c$. Then differentiating with
respect to $T_c$ and setting the result equal to zero, one
can show that the optimum $T_c$ for maximum efficiency is,

$$
(T_c)_{\text{opt}} = 1 + (\gamma') \frac{1}{2} \left[ 1 - \left( \frac{B - D}{D} \right) \right]^{1/2}
$$

From equation (45), one can obtain limiting boiler
temperature for the real solution of $(T_c)_{\text{opt}}$. Fig. 6
shows the plots of $(T_c)_{\text{opt}}$ vs $\gamma'$ with $(B-1)/D$ as a
parameter. Thus

$$
\left( \eta_{\text{net}} \right)_{\text{opt}} = (B - D) - (D \gamma' + 2 (D \gamma')^{1/2})
$$

$$
[1 - (B + D) \gamma']
$$

Fig. 6b gives the characteristics of equation (46).

4. SOLUTIONS WITHOUT ASSUMPTION $T_e = T_e$

In this section the assumption that $T_e = T_e$ is re-
laxed to include the effects of finite heat transfer
rate. Optimizations are carried out for frontal areas
and efficiency based on useful power output. For the
present case using simple heat transfer relations, the
exit air temperature may be shown to be given by:

$$
(T'_c - T_a') = (T'_c - T_a') \left[ 1 - \exp\left( -A' \phi \beta C' / \omega \right) \right]
$$

The method of derivation is the same as in Section 3.1.
In addition to transformation given in Section 2.3 the
following transformations are introduced.

$$
x = x'x
$$

where $x' = \left( [v^2 \gamma y] / (\gamma' y - 1) \right)^{(1-m)/m}$,

$$
\beta = \frac{\phi \alpha}{1 - \phi \alpha}
$$

where

$$
\phi = \frac{2}{1 - \exp \left( (1-m)/m \right)}
$$

and

$$
\alpha = \frac{1}{\eta \left( \frac{T_c - 1}{}ight)}
$$

Notice that, for small $(1-m)/m$, $\phi = \gamma^2$. Also there
are three independent parameters $x$, $v$, and $T_c$. The term
$\phi$ is defined as the cooling fan loading factor.

4.1 Frontal Area

By a similar procedure as in Section 3.1 and with the
assumption that $A_1 = A_0 = A$, one can show that

$$
A = \frac{(\alpha - \alpha)}{(1 - \phi \alpha)} \frac{1}{V} \frac{1}{(1 - e^{-x^{-1-m} \gamma})}
$$

In order that $A$ be an absolute minimum with respect to
$x$, $v$, and $T_c$,

$$
\frac{\partial A}{\partial x} = \frac{\partial A}{\partial v} = \frac{\partial A}{\partial T_c} = 0
$$

Thus carrying out differentiation on equation (51) and
making use of equation (52), the following equations
are obtained.

$$
\frac{\partial A}{\partial x} = 0
$$

$$
e^{-x^{-1-m} \gamma} = \alpha V
$$

For the case,

$$
\frac{\partial A}{\partial x} = 0
$$

$$
\frac{\partial A}{\partial v} = 0
$$

Thus solving,

$$
\left( \frac{\partial A}{\partial v} \right)^{1/2} = 0
$$

Thus for absolute optimization,

$$
\left( \frac{A}{\alpha (x)} \right)^{1/2} = 4.58
$$

Using equation (55a) in (49b) and (49a),
Thus one is looking for $\alpha_{\text{min}}$ for the absolute optimisation of $A$.

$$A_{\text{min}} = 4.58 \, (\alpha \sigma)^{1/2}$$

Using the relation for $\alpha$ as given by equation (22), $\alpha_{\text{min}}$ is given by equation (25).

4.2 Actual Efficiency

Following the same procedure as outlined in Sec. 3.4,

$$T_{\text{c}}^{(\text{opt})} = 1 + (\phi)^{1/2} \left(1 - \frac{B - 1}{D}\right)^{1/2}$$  \hspace{1cm} (56)

$$\eta_{\text{net}}^{(\text{opt})} = \frac{(B - D) - \left[D \phi + 2(D\phi)^{1/2}
\left(1 - B + D\right)^{1/2}\right]}{B - D}$$  \hspace{1cm} (57)

Figs. (6a) and (6b) still hold good for these equations with the definition for $\phi$ given by equation (49b).

5. RESULTS

The results will be discussed with a numerical example. Consider an engine developing 75 kW of useful power operating on a Rankine cycle using water as the working fluid with the boiler pressure at 58 bars. The cooling medium is ambient air at 27°C blown by a fan over the condenser. It is required to optimise the front areas, heat transfer areas and efficiency. Assume $\eta = 1/5$, $a = 0.072$, $y = 1.4$, $Pr = 0.7$, $P = 1$ bar, $u = 1.9 \times 10^{-5}$ m²/sec, and $T' = 94^\circ$C.

5.1 Case (a): $T' = T_e^*$

First, efficiencies for various $T_c$ are chosen from Fig. 2, the values for $\alpha$ are estimated from Fig. 3 and the $\alpha$'s are replotted in Fig. 7 with $T_c$ as the independent coordinate. On the same graph, efficiency is also plotted. Thus $\alpha_1$, $\alpha_2$, $\alpha_3$, $\alpha_4$, and $\alpha_5$ are determined from Fig. 7. Then using equation (26), (10b), (11a), (11b) and (22), the following results are obtained.

For minimum inlet area, $\alpha_1 = 7.2$, $T_c = 1.36$, $\eta = 0.27$, $A^* = 70.8$, $v^* = 3.4 \times 10^{-6}$ m²/$\text{sec}$, $v^* = 620$ m/s and $V^* = 94$ m/s. Also $T_c^*$ is about $135^\circ$C. One can compare this with optimum condenser temperature of about $100^\circ$C obtained by Vickers et al. [1] for minimum condenser size for a Rankine cycle with boiler pressure as 54 bars. Table III lists the results for minimum outlet area and minimum heat transfer area (with $m = 472$/m etc.). It is observed from Table III that for the optimisation of condenser size, one has to trade off net engine efficiency. For the optimum efficiency $\phi$ is assumed to be about 0.05. However, reasonable value for $\phi$ could be obtained for the condition that condenser size is minimum with respect to velocity of the air.

5.2 Case (b): $T' \neq T_e^*$

Again making use of the solution in Section 4.1 and 4.2, $\alpha_{\text{min}} = 7.2$, $T_c = 1.36$, $A^* = 88.5$, $A^* = 0.03$ m², $v = 1.44$, $v' = 89$ m/s, $x = 1.381$, $x^* = 0.6$ m, $x^* = 0.81$ m and $v' = 89.0$ m/s. Table IV compares the results for both cases $T_c^* = T_e$ and $T_c^* \neq T_e$.
Figure 1. Configuration of Fins Around Condenser

Figure 2. Rankine Efficiency vs. Condenser Temperature

Figure 3a. $a_i$ and $a_e$ vs. Condenser Temperature with Rankine Efficiency as a Parameter

Figure 3b. $\alpha_0/\alpha_i$ vs. Condenser Temperature with Rankine Efficiency as a Parameter
Figure 4a. $A_i/a_i/v_i$ and $A_0/a_0/v_0$ vs. $V\sqrt{2a_i}$

Figure 4b. $\tilde{A}_w$ vs. $V\sqrt{2a_i}$ with $m$ as a Parameter

Figure 5. Dimensionless Heat Transfer Area vs. Dimensionless Volume when Space is Limited
Figure 6a. Variation of Optimum Condenser Temperature with Cooling Fan Loading Factor $\dot{\phi}$ and $\phi$

Figure 6b. Variation of Optimum Efficiency with $(D\phi)^{1/2}$ and $(D\phi)^{1/2}$
Figure 7. Rankine Efficiency $\eta$, $a$, $a_o$, and $a_o/a_i$ vs. Condenser Temperature

Table 1. Optimisation Studies

<table>
<thead>
<tr>
<th>No.</th>
<th>Variable Optimised</th>
<th>with respect to</th>
<th>Assumptions</th>
<th>Fixed Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Front Area</td>
<td>$T_C$</td>
<td>a) Exit air temperature equals the condenser temperature and $T_C$</td>
<td>a) Engine useful power ($\dot{W}_u$) b) Air velocity</td>
</tr>
<tr>
<td>2.</td>
<td>Front Area</td>
<td>$V'$</td>
<td>a) $\dot{W}_u$ b) $T_C$</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>Total heat transfer area or condenser volume</td>
<td>$T_C$</td>
<td>b) any Rankine cycle</td>
<td>a) $\dot{W}_u$ b) $T_C$</td>
</tr>
<tr>
<td>4.</td>
<td>Total heat transfer area or condenser volume</td>
<td>$V'$</td>
<td>a) $\dot{W}_u$ b) $T_C$</td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>Useful net thermal efficiency</td>
<td>$T_C$</td>
<td>a) $\dot{W}_u$ b) Cooling fan loading factor</td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>Frontal Area</td>
<td>$T_C$</td>
<td>Any Rankine Cycle</td>
<td>a) $\dot{W}_u$ b) $V'$</td>
</tr>
<tr>
<td>7.</td>
<td>Frontal Area</td>
<td>$V'$</td>
<td>Any Rankine Cycle</td>
<td>a) $\dot{W}_u$ b) $T_C$</td>
</tr>
<tr>
<td>8.</td>
<td>Useful net thermal efficiency</td>
<td>$T_C$</td>
<td>Any Rankine Cycle</td>
<td>a) $\dot{W}_u$ b) Cooling fan loading factor</td>
</tr>
</tbody>
</table>
Table II. Coefficients in Efficiency Fit for Rankine Cycle

\[ n = B \cdot D \cdot T_c, \ D = \frac{T_c}{T_B}, \ T_B = \frac{T_g}{T_B} \]

T_B = saturation temperature corresponding to the boiler pressure

<table>
<thead>
<tr>
<th>Peak Pressure (bar) (psia)</th>
<th>B</th>
<th>D</th>
<th>D_G</th>
</tr>
</thead>
<tbody>
<tr>
<td>68 (1000)</td>
<td>1.10086</td>
<td>0.59057</td>
<td>1.1100</td>
</tr>
<tr>
<td>61 (900)</td>
<td>1.100306</td>
<td>0.60721</td>
<td>1.1130</td>
</tr>
<tr>
<td>54 (800)</td>
<td>1.10878</td>
<td>0.61835</td>
<td>1.1200</td>
</tr>
<tr>
<td>48 (700)</td>
<td>1.11090</td>
<td>0.62899</td>
<td>1.1230</td>
</tr>
<tr>
<td>41 (600)</td>
<td>1.14595</td>
<td>0.66793</td>
<td>1.1660</td>
</tr>
<tr>
<td>34 (500)</td>
<td>1.15129</td>
<td>0.68380</td>
<td>1.1700</td>
</tr>
</tbody>
</table>

Table III. Operating Conditions for various optimised parameters of a Condenser for an Automotive Steam Vehicle

<table>
<thead>
<tr>
<th>Quantity</th>
<th>( T_c )</th>
<th>( n_{net} )</th>
<th>( \nu' )</th>
<th>( A_1 )</th>
<th>( A_0 )</th>
<th>( A_w )</th>
<th>( n_f )</th>
<th>( n_c )</th>
<th>( V_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak pressure</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

*\( \delta = 0.05 \) is assumed.

Table IV. Comparison between the results for case a: \( T_c' = T_e' \) and case b: \( T_c' \neq T_e' \)

<table>
<thead>
<tr>
<th>Parameter to be optimised</th>
<th>( T_c' = T_e' )</th>
<th>( T_c' \neq T_e' )</th>
<th>( n_{net} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frontal Area</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( T_c' = T_e' )</td>
<td>89</td>
<td>0.81</td>
<td>0.0994</td>
</tr>
<tr>
<td>( T_c' \neq T_e' )</td>
<td>94</td>
<td>1.24</td>
<td>0.0796</td>
</tr>
<tr>
<td>Efficiency</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( T_c' = T_e' )</td>
<td>-</td>
<td>-</td>
<td>-.</td>
</tr>
<tr>
<td>( T_c' \neq T_e' )</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

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