A CORRELATION OF COMBUSTION EFFICIENCY
WITH DIÄMKÖHLER NUMBER FOR FLUIDIZED BEDS

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ABSTRACT

This paper describes a model for correlating all the past laboratory type experimental data for the char combustion efficiency with a new dimensionless group called Däkmöhlzer number $V$ when FBC (Fluidized Bed Combustors) is fired with solid fuel particles of narrow size range. Essentially Däkmöhlzer number $V$ is a ratio of residence time available for char in the bed to the reaction time. Total combustion efficiency can also be correlated with $D_V$ as long as volatiles combustion is almost complete.

First analysis is carried out by treating the bed as a two phase reactor (TPR). Volatiles combustion is treated as kinetically controlled burning with a single step reaction of an arbitrary order. For char combustion, the size distribution is obtained from mass conservation; then the weight of the char in the bed and the burn char fraction are obtained. For any given bed temperature, volatiles and char combustion efficiencies can be determined. With use of energy conservation equations, the bed temperature is also determined, which reveals the extinction condition of the bed.

From the two phase reactor analysis and the kinetics of char oxidation, solutions for nondimensional bed weight and char combustion efficiency can be obtained in terms of Däkmöhlzer number $V$. The theoretical weight of char in the bed is compared with experimental results reported elsewhere as a check on the present modelling approach. Further a method is shown where it is possible to detect the mechanism of diffusion or kinetics controlled burning of char from the measured weight of char.

The universal correlation for char combustion efficiency is compared with the past experimental data collected from the open literature for fluidized beds operated under various conditions. The correlation appears to be very good. Based on char combustion correlation, it appears that Däkmöhlzer number $V$ should be higher than 2 to 3 in order that char combustion efficiency is at acceptable levels. This Däkmöhlzer number $V$ can serve as the scaling group when FBC is scaled up to a higher thermal rating.

INTRODUCTION

Fluidized Bed Combustion is now emerging as a promising technology for firing solid fuels of widely varying heating values and at the same time burning those fuels in an environmentally acceptable manner. Typically a fluidized bed combustor (FBC) involves burning of any fuel as long as its heating value is more than the sensible energy at stable bed temperature (Kapp, 1979; Anson, 1976; Raaf, 1973). Unlike conventional combustion systems where heat release due to chemical heat release occurs within a narrow volume, the heat release in FBC occurs within and possibly above the bed. Further heat exchangers are located within heat release zone as opposed to conventional boilers. Thus the suspended particle in conventional burners is near adiabatic flame temperature attendant with high reaction rate and diffusion controlled burning while for fluidized beds, particle temperature is near bed temperature (1000 - 1200 K) with burning possibly limited by kinetics of reaction.

The operation of fluidized bed combustors at low temperature and excess air coupled with the use of alternate fuels with low heating values (such as biomass, toxic waste, cotton gin trash, and feedlot manure) can lead to the problem of extinction of bed in the low temperature range of the bed and hence to the load control (Annamalai, 1979; Annamalai, 1983). Thus operating variables such as $AP$ ratio, air velocity, heat removal rate (load), etc., must be closely controlled in order that the flame extinction (thermal extinction) in the bed does not occur. The recent testing of combustion of high volatile manure in FBC reveals similar results (Sweeten, 1981). The thermal performance and extinction phenomena including correlation of bed combustion efficiency and bed char weight with a new nondimensional group called Däkmöhlzer number $V$ will be dealt in the present paper.

STATEMENT OF THE PROBLEM

Thermal performance of FBC is analyzed using two phase reactor (TPR) theory (Campbell, 1975). The model includes kinetics controlled burning of char and volatiles with heat loss via in bed heat exchangers and as sensible heat loss via exhaust. Also a method is given whereby kinetics of char combustion can be extracted from the measured weight of char in the bed. Finally, the analysis carried out is such that the combustion efficiencies for all fluidized bed combustors can be correlated with Däkmöhlzer number $V$. Past experimental data are collected and correlated with this group.

LITERATURE REVIEW

There exist various kinds of fluidized bed combustor models with differing complexity and viewpoint which are published in the open literature. They have been quite recently listed (Funk, 1980). The heat generated in the bed is due
to kinetics limited burning of volatiles and char while heat is lost as sensible heat and as the energy transfer to cooling water. Bed temperature is thus controlled by heat generation and heat loss rates. The thermal theory of combustion leads to multiple solutions for the burning rate leading to ignition and extinction condition. Jain and Mukinda (1967) studied the ignition and extinction regimes for gaseous flames. Sibulkin et al studied the extinction problem of vertically burning fully pyrolyzing solid surfaces (1982). Annamalai and Venkata (1979) assumed complete combustion of char and using perfectly stirred reactor (PSR) theory, and kinetics limited combustion of volatiles, they obtained preliminary results on extinction and turn down ratio (i.e. ratio of maximum flow rate of fuel to minimum flow rate of fuel at extinction) attainable in the fluidized bed combustor.

Gordon et al (1976,1978) studied the problem of burning of char/carbon particles which is assumed to be completely burnt to CO (i.e. no elutriation) with CO combustion limited by kinetics of oxidation. Congalidis et al (1981) studied the multiplicity patterns in FBC with assumption of complete combustion of volatiles to CO. and kinetics controlled char burning to CO followed by kinetics controlled burning of CO to CO₂.

Rajan and Wen (1980) assumed that the volatiles burn to CO in the emulsion phase and to CO in the bubble phase. Char burns to CO/CO₂, which subsequently oxidizes to CO. Wells et al (1980) assumed that volatiles burn instantaneously to CO and char burns to CO followed by oxidation in both the emulsion and bubble phase. Annamalai (1983) assumed that kinetics controlled burning of volatiles to CO, and char to CO₂ and obtained thermal performance of fluidized bed combustor.

MODEL AND ASSUMPTIONS

The present modelling work was undertaken in order to present a single scaling group with which the experimental results for combustion efficiency can be correlated for all fluidized bed combustors. Such group is called as Damköhler number V.

The model includes kinetic controlled burning of both char and volatiles with char burning to CO/CO₂ while volatiles burning to CO/CO₂. Generalized results for weight of char in the bed and char combustion efficiency for all FBC can be correlated with Damköhler number V.

The bed is classified into different regimes of combustion: i) TRR (two phase reactor) ii) CMR (completely mixed reactor) and iii) PSR (perfectly stirred reactor). The terms CMR and PSR need classification, if the crossflow between the bubble and the emulsion phases is such that the mass fraction leaving the bubble is almost same as the emulsion oxygen mass fraction, then the reactor can be assumed to be completely mixed reactor (CMR) even though the bed is still a two phase reactor. If the exchange coefficient is extremely high such that there is uniform gaseous concentration both in emulsion and bubble phases throughout the bed, then the bed is assumed to be a perfectly stirred reactor (PSR).

MODEL

Essentially two phase reactor (TPR) theory will be utilized for the analysis. CMR and PSR results can be obtained as an extension of TPR results. Later it is shown that the results obtained for PSR and CMR can be converted to results for TPR with suitable scale factors. As reviewed earlier first approximate equations are to be developed for i) the heat generation in the bed due to oxidation of volatiles and char, ii) the heat loss to the heat exchangers and iii) the sensible heat loss (Figure 1.4). General approach to the analysis is as follows. Bubbles carrying bulk of oxygen transfers oxygen to the emulsion phase in which both volatiles and char burn and consume the oxygen. A fraction of the heat generated by the oxidation of the fuel is supplied to the heat exchangers while the remaining is supplied as sensible heat to the gases and to the circulating solids. Thus the species and energy balance equations yield the solution to the bed temperature as a function of feed and fuel parameters. General assumptions are given in the next section. More assumptions will be stated as the analysis is carried out.

ASSUMPTIONS

1) Volatiles are liberated immediately in the emulsion phase. 2) The emulsion phase is assumed to be well stirred. 3) The mass fraction of CO, volatiles, and oxygen in the emulsion phase are dictated by the oxidation and production rates of CO, and volatiles and by gas exchange between bubble and emulsion phases. 4) The mass fraction of CO, volatiles, and oxygen in the bubble phase are influenced by oxidation rates in bubble phase and by cross exchange between the bubble and emulsion phases and the input feed rate of air through the bubble phase. 5) The bubble phase is treated as a plug flow region and reaction is frozen in the bubble phase. Even if reaction takes place in bubble, this can be accounted by adjusting the volume of reaction zone for the volatiles (not necessarily equal to the volume of bed unless exchange rate is high). Further PSR theory can account for reaction in both phases. The exchange factor between bubble and emulsion phases is allowed to vary along the bed. 6) Only two kinds of reactions take place. (See Figure 1. b.) 7) The carbon burnout proceeds under constant density and the particle radius is allowed to change according to the power law: \( R_i = R_i^{b_i} - \alpha t \). 8) The temperature difference between the particle and the bed is assumed to be small. The implication is as follows:

\[
\alpha_t = \beta_1 \exp(-\frac{T}{T_b^{\alpha_1}}) \frac{p_{CH_4}}{T_b^{\alpha_2}}
\]

If \( \Delta T = T_1 - T_b \), then \( \alpha_t \approx 1 \)

\[
\alpha_t = \beta_1 \exp(-\frac{T}{T_b^{\alpha_1}}) \exp[2\gamma(\Delta T/T_b^{\alpha_2})] \frac{p_{CH_4}}{T_b^{\alpha_2}}
\]
Modifying the preexponential factor, equation (2) is written as

$$h^* = B \exp(-E/R^T/K_b^2) \cdot \text{ch} V_0$$

where

$$B = B_0 \exp(E/RT)$$

(3)

As $\Delta T \to 0$, $B = B_0$. If $\Delta T/\Delta T_p = 0.2$, $E = 67,000$ KJ/Klnole, $T = 1100 K$, $B=4B_0$. The implication of change in $h^*$ factor will be discussed later. It should be noted that for smaller sized particles, the convective heat loss is very high while for the larger sized particles, the radiant heat loss is high and hence $T$ is normally small. Further, the heat loss rate under highly agitated conditions in the bed tends to make the particle temperature almost same as the bed temperature. 9) Ejection and entrainment rates are assumed to be proportional to the bed weight and inversely to a power function of particle diameter.

$$K'(N) = \frac{h^*}{h}$$

(5)

DERIVATION OF RESULTS

VOLATILE BURNT FRACTION

We can formulate the oxygen transfer from the bubble phase to the emulsion phase as the following:

$$K_{bb} \cdot \left( Y_{0,b} - Y_{0,e} \right) = u_b \cdot \left( \frac{dY_{0,b}}{dy} \right)$$

(6)

The interchange coefficient between the bubble and the emulsion phase, $K_{bb}$, and bubble velocity, $u_b$, are calculated by the use of equations described in Kunii and Levenspiel (1969).

Defining

$$Z = \int_0^Y \left( K_{bb}/u_b \right) dy$$

and using equation (7) in equation (6) and with assumption 2, we can obtain the following equation for oxygen:

$$\frac{Y_{0,e} - Y_{0,i}}{Y_{0,e} - Y_{0,i}} = \exp(-Z)$$

(8)

where $Y_{0,e}$ is equal to $Y_{0,i}$ at $Z = 0$.

With this, similar procedures for volatiles,

$$\frac{Y_{v,e} - Y_{v,i}}{Y_{v,e} - Y_{v,i}} = \exp(-Z)$$

(9)

where $Y_{v,e}$ is equal to $Y_{v,i}$ at $Z = 0$.

Also

$$\dot{M} Y_{0,i} = \dot{M} Y_{0,e} + \dot{M}_b Y_{0,b}$$

(10)

and

$$\dot{M}_m = \dot{M}_e \cdot T + \dot{M}_b \cdot R_b$$

(11)

where

$$\dot{M}_m = A \cdot u_{mf} \cdot o_i$$

$$\dot{M}_b = A \cdot \left( u - u_{mf} \right) \cdot o_i$$

(12)

where $T_m$ is the average temperature of the bubble and emulsion phase and generally varies with the height of the bed. Average oxygen concentration at any point in the bed can be given by substituting $Y_{0,b}$ from equation (8) in equation (10)

$$Y_{0} = Y_{0,e} \cdot M + (1 - M) \cdot Y_{0,i}$$

(14)

where the mixing factor between the bubble and emulsion phases is given as

$$M = 1 - \left( u - u_{mf} \right) \cdot \exp(-Z)/\mu$$

(15)

Similarly,

$$\left( T_m - T_1 \right) = \left( T - T_1 \right) \cdot M$$

(16)

Note that if $M$ is close to unity when $Z = 0$, then $Y_{0,e}$ is equal to $Y_{0,i}$ and $T_m = T$ and this reactor under these conditions is defined to be completely mixed reactor (CMR) even though the reactor is still a two phase reactor. Similarly for volatiles,

$$Y_{v} = Y_{v,e} \cdot M + (1 - M) \cdot Y_{v,i}$$

(17)

The measured average mass fraction of $Y_{0,e}$ can be utilized to obtain the best average value for $M$ and hence $K_{bb}/u_b$ from eqs. (15) and (7). Note that if exchange coefficient $K_{bb} \rightarrow 0$, $K_{bb}/u_b \rightarrow 1$, $M \rightarrow 1.0$ (perfectly stirred reactor, PFR). On the other hand, for slow bubbling regime $u \rightarrow u_{mf}$, $M \rightarrow 1.0$ but for the slow bubbling regime, the assumption of uniform $Y_{0,e}$ may not be valid. The difference between $Y_{0,e}$ and $Y_{0,i}$ is due to the consumption of oxygen by oxidation of volatiles liberated from solid fuel particles and oxidation of char particles. Hence

$$M \cdot Y_{0,i} - Y_{0,i} = f_v \cdot R_v \cdot Y_{v,v} + f_{ch} \cdot R_{ch} \cdot Y_{v,c}$$

(18)

For single step reaction of volatiles with $O_2$, the volatiles burnt fraction is given as

$$f_v = R_v \cdot K_v \cdot Y_{v,v} / \dot{M}_v$$

(19)

where $K_v = R_v \cdot \exp(-E_v/R_T)$
Similarly for volatiles

\[ Y_{v,i} = \frac{Y_{v,H}}{Y_{v,e}} = f_{v} \]  

(21)

First use equation (17) in equation (21) to eliminate \( Y_{v,H} \) and then solve for \( Y_{v,e} \) for assumed \( Y_{v,e} \) and temperature. Then use the resulting equation for \( Y_{v,e} \) in equation (18) and equation (14) for \( Y_{O,e} \) to solve for \( Y_{O,e} \) and \( f_{v} \) at all \( \phi \) given temperature, \( \theta \).

The final equation for \( Y_{o,e} \) is shown below.

\[
( Y_{o,i} = Y_{O,e} ) = D_{i} \exp \left( -E_{0}^{*} / \theta \right) \frac{Y_{O,e}^{n_{o}Y_{O,e}}} {Y_{O,e}^{n_{o}Y_{O,e}}} \left( 1 + D_{i} \frac{Y_{O,e}^{n_{o}Y_{O,e}}} {Y_{O,e}^{n_{o}Y_{O,e}}} \right) \]

(22)

where

\[
Y_{o} = Y_{o}/M_{o} \]  

(23a)

\[
\theta = T/T_{1} \]  

(23b)

\[
D_{i} = S_{i}^{*} \frac{M_{o}}{M_{c}} \]  

(23c)

\[
D_{i} = S_{i}^{*} \frac{M_{o}}{M_{c}} \]  

(23d)

Note that Eq. (22) does not contain \( M_{c} \) explicitly. Further \( M_{c} = 1 \) for CMG and \( M_{c} = 1, H_{c} = H \) for PSR theories.

**WEIGHT OF CHAR AND CHAR BURN END FRACTION**

**Mass Balance Equation**

The coal particles fed into the bed consist of distributed size ranges of particles. The simplest case is to treat as monosized streams. If we know the input feed size distribution, then the input particles can be classified into a number of monosized streams and char burn fraction should be calculated as a sum of char burn fractions of each individual monosized streams. The combustion of these particles takes place in the emulsion phase. Some of the particles in the bed are elutriated and the others continue to burn. Kunii and Levenspiul (1969) has given a generalized differential equation to determine the size distribution in the bed.

The mass balance equation is expressed in the following mathematical form as follows (Kunii, 1969)

\[
P_{0} P_{1}(R) - F_{1} P_{1}(R) = K(R) W_{1}(R) - W_{0}(R) P_{1}(R) - \frac{\partial}{\partial t} \left[ P_{1}(R) (\partial R / \partial t) \right]
\]

(24)

Rewrite

\[
F_{1} P_{1}(R) = F_{1} P_{1}(R) W_{1}/W
\]

(25)

and let

\[
K'(R) = K(R) + F_{1}/W
\]

(26)

where \( K' \) represents the particle loss coefficient due to the elutriation and overflow.

Also,

\[
\int_{0}^{\infty} F_{1}(R) dR = 1
\]

(27)

Furthermore, integration of equation (24) leads to the overall mass balance equation.

\[
F_{0} - \int_{0}^{\infty} K'(R) W_{1}(R) dR = - \int_{0}^{\infty} \left[ \frac{\partial}{\partial t} \left[ P_{1}(R) (\partial R / \partial t) \right] \right] dR
\]

(28)

The total char burning rate, \( R_{ch,b} \) is given as

\[
R_{ch,b} = \int_{0}^{\infty} \left[ \frac{3}{2} P_{1}(R) (\partial R / \partial t) \right] dR
\]

(29)

The char burn fraction \( f_{ch} \) defined as a ratio of total char burning rate to the char flow rate.

\[
f_{ch} = \frac{R_{ch,b}}{R_{c}}
\]

(30)

Once \( P_{1}(R) \) (the size distribution of char in the bed) and \( W \) (the total weight of the bed) are determined, the char burn fraction can be determined. The elutriation rate can be determined by the following equation.

\[
K(R) = N R^{n}
\]

(31)

where value of \( q \) is -1.65, and for Osberg and Charleworth (1951), \( q = -2 \). For the case that char particle concentration is larger than 1%, \( N \) is given by the following equation (Wen, 1982).

\[
N = 9.43 \times 10^{-4} \left[ \frac{A_{c}}{A_{w}} \right] \left[ \frac{u}{v} \right]^{2.65}
\]

(32)

The generalized relation between the char radius \( R \) and time \( t \) is given by the following equation.

\[
R^{n} = R_{0}^{n} - at
\]

(33)

where \( n = 2 \), if the burning of char takes place under diffusion control.

\( n = 1 \), if the burning of char takes place under kinetic control with constant density, and under attrition.
ZrCl₄, if the burning of char takes place under mixed control.

From equation (26) and (31), we can get

\[ K' = \frac{k}{W} \]  

Note that if there is no elutriation, \( N = 0, m = 0 \), then \( \phi = \frac{F_r}{W} \), and if there is no overflow, \( F_1 = 0, m = \varrho \), then \( N = \vartheta \). If solids are returned back to the bed completely, then \( \varphi_1 = -\frac{W}{W} \) and \( \varphi = 0 \).

Size Distribution

Equation (24) can be normalized using the parameters defined below:

\[ r = \frac{t}{t_{\text{ref}}} \quad t_{\text{ref}} = W/P_0 \quad \varrho = W/R \]  

\[ \varphi = P_0(r)dr = \frac{P(r)}{P_0}dr \]  

\[ \alpha = t_{\text{ref}}R \quad \frac{1}{D_0} = \frac{1}{D_\varphi} = \frac{n_m}{n_m} \frac{\varphi}{\alpha} \]  

(35)

where \( D_0 \) is Dinkilhler number V and \( D_\varphi \) is the inverse (reverse) of Dinkilhler number \( V' \).

We can get the following normalized equation by introducing equations (31), (33), (34) and above parameters listed by eq. (35) into equation (24).

\[ \frac{dP_0}{dr} = \left[ \frac{3}{\alpha} + D_\varphi \frac{r^{n_m+1}}{n_m} \right] 
- P_0(r) \]  

(36)

Equation (36) is a nonhomogeneous, linear, first order differential equation and can be integrated for the given input coal size distribution with the following relationship.

\[ P_0(r = 1) = 0 \]  

(37)

where \( \delta \) is the dirac-delta function.

Integrating equation (24) and using eq. (27), the size distribution in the bed is given as follows:

\[ \varphi = \frac{(\omega/\omega_0)^{n_m}}{D_0 + 3 + n} r^{3 + n} \]  

if \( n_m = 0 \)  

\[ = \frac{(\omega/\omega_0)^{n_m} r^3 \exp(D_\varphi r^{n_m}/[n_m])}{(\int_0^\infty 2 \sqrt{n_m} \exp(D_\varphi r^{n_m}/[n_m])dr)} \]  

if \( n_m = 0 \)  

(38)

(39)

If \( D_0 = 0 \), i.e., no particle loss by elutriation, equation (38) states that the mass of particles in the bed per unit radius is proportional to the 4th power of particle radius for diffusion controlled burning that agrees with the results of Campbell and Davidson (1975).

Weight Of Char In The Bed

The weight of char in the bed can be determined by using equation (27), (38), (39). The results are given below.

\[ W = \omega/\omega_0 \varphi \]  

\[ = \frac{1}{D_\varphi} \int_0^\infty 2 \sqrt{n_m} \exp(D_\varphi r^{n_m}/[n_m])dr \]  

(40)

where

\[ \varphi_1 = \int_0^\infty 2 \sqrt{n_m} \exp(D_\varphi r^{n_m}/[n_m])dr \]  

(41)

\[ \varphi_2 = \int_0^\infty 2 \sqrt{n_m} \exp(D_\varphi r^{n_m}/[n_m])dr \]  

(42)

\[ \varphi_3 = \int_0^\infty 2 \sqrt{n_m} \exp(D_\varphi r^{n_m}/[n_m])dr \]  

(43)

Char Burnt Fraction

The previous results for dimensionless weight (equation (40)) and size distribution (eqs. (38) and (39)) can now be utilized in eq. (27) to derive the results for the burnt char fraction.

\[ \frac{f_{\text{ch}}}{f_{\text{ch}}} = \frac{\omega/\omega_0}{D_\varphi (f_2 - 3f_3)} \]  

(44)

where \( f_2 \) and \( f_3 \) are given by eqs. (42) and (43). Equation (44) is relatively simple and for given \( m \) and \( n \) the results for the burnt fraction can be correlated with \( D_\varphi \) or Dinkilhler number \( V' \) for all fluidized bed combustors. Particularly for low volatile solids like anthracite, the total combustion efficiency can be correlated with this number.

Given the bed temperature, one must first determine \( \varrho \) from eq. (22) and then use the oxygen mass fraction to determine the reaction rates of char particle (see eq. (33)). Knowing the reactivity \( \alpha \), the Dinkilhler number \( V' \) is evaluated from eq. (35) and then the weight of char and char combustion efficiency are determined from eqs. (40) and (44). Experimental data are normally available for both the bed temperature and char combustion efficiencies. The knowledge of bed temperature thus enables us to calculate Dinkilhler number \( V' \) and then correlate the char combustion efficiency with \( D_\varphi \).

The results so far obtained are sufficient for correlating the experimental data with Dinkilhler number \( V' \).

* The solution obtained by this method is called Green's function.
For monosized feed, the nondimensional weight of char in the bed can be determined using equation (40), and the results are plotted in the Figure 2. It is apparent that the given n, a change in m (elutriation relation) does not significantly affect the dimensionless weight. If feed size distribution is known, dimensionless weight in each size range can be determined by knowing F in each size range and using Figure 2. Then the dimensionless weight for the given input size distribution can be determined by summing up each dimensionless weight.

Special cases are discussed below.

Case (i) n = 0 or D

With n = 0 in eqs. (40) - (43), the following relation is obtained:

\[ W = x W_0 F_R^0 \]  

(48)

Note that n = 2, 1 for diffusion and kinetics control respectively and such results eq. (48) are applicable to both mechanisms if m = -2, -1 respectively. Figure 3 shows the results for weight under diffusion control burning with m = -2 (curve AB).

Case (ii) n = 1

Again using eqs. (40) - (43)

\[ W = x W_0 F_R^0 = E_3 \frac{D_s}{D_1} \]  

(49)

Where B's are exponential integrals.

The results check with Kali and Lovenru (1969) in that as D

\[ \to D_0 \to \infty \]  

i.e. no particle loss, particle residence time \[ \to \infty \], dimensionless weight \[ \to 0.25 \]. Again if n = 1, and m = -2, equation (49) is applicable for kinetics controlled burning or for particle drainage under attrition and abrasion processes. (See curve CD in Figure 3 for D

\[ \to D_0 \]  

with m = -2 under kinetics controlled burning.) Figure 3 also plots W

\[ D_0 \]  

where W

\[ = \text{compared to} \]  

the curves EF and GH are useful if particle loss rate is fixed and the reactivity of the particle is altered. Thus as burning rate constant is decreased (D

\[ \to D_0 \]  

is increased), the weight of the bed increases for the same feed rate of char, number of particles must increase in order to attain the total burning rate of all particles as same as input feed rate less the particle loss rate. The curve AB and CD with dotted lines are useful if reactivity of char is known and if particle loss rate is altered. Then as particle loss rate constant is increased, the particle loss rate increases; for the same input feed rate less amount remains to be burnt. Since reactivity of each char particle is fixed then there should be less number of char particles and hence less weight remains in the bed. For B/C operating with particles returned to the bed \[ \approx \]  

can be set equal to zero (see eq. (34) i.e. D

\[ \to D_0 \to \infty \]  

). By knowing the weight of the char in the bed, input feed rate, input feed size, the reactivity \( a \) can be determined since \( W \) \[ = W_0 \frac{F_R^0}{\gamma} \] approaches 0.25 for kinetics controlled burning as \[ \to \infty \]. If one plots \[ W_0 / (W_0 + \gamma) \] vs. \( 1/T \) at fixed \( D_0 \) or at fixed exit oxygen...
concentration (for example, the experimental results of Donat et al. (1980)) the overall activation energy for kinetics controlled burning process can be obtained. It should be cautioned that there may be mixed control for certain size ranges. Thus kinetic constants evaluated are only global values. On the other hand, if measured k vs. 1/T indicates a linear increase, the burning process in the bed is a diffusion controlled process.

A check on the results for weight can be obtained by comparing the theoretical results based on the experimental data of Donat, Massimilla et al. (1980) for South African Coal. CPR theory was assumed to be valid (M = 1). The results are shown in Figure 4 for two FBC temperatures. For film air flow, coal feed rate was reduced which increased the oxygen concentration in the exhaust and reduced the weight in the bed. Thus the qualitative trends of the experiment are confirmed by the theory. Use of CPR theory may only result in lower bed weight since the equilibrium oxygen concentration will be lower and which results in lower burning rate for the TFR case. Apart from mixedness parameter N, the other two adjustable constants which can alter the weight are plugging rate constant (with no overflow) and reactivity of char particles. But qualitative trends are not expected to change.

BURNED CHAR FRACTION

General Results

Equation (44) is used to obtain the burnt char fraction. Figure 5 plots the results for burnt char fraction with (n, m) as a parameter. Some special cases where explicit solutions for \( f_{ch} \) are possible are given below.

Diffusion Controlled Burning

For this case, \( n = 2 \) and hence from equation (44) and with \( m = -2 \),

\[
\frac{f_{ch}}{1 - (24/330)} = \frac{L}{1 + (24/330)}
\]

(50)

The curve AB in Figure 5 shows the results for burnt char fraction. Note that (i) as \( n \to 0 \), the initial size \( R_e \), becomes unity and (ii) for any \( n \), the burnt char fraction is independent of the initial size \( R_e \). Thus the results for burnt char fraction are true whether the input fuel is of monodispersed or distributed size. The reason is that while the time increases as the square of the size of the particle, the residence time in the bed also increases as the square of the particle size since elutriation rate decreases quadratically with increase in particle size.

The constant \( a_2 \) is determined from the analysis of diffusion controlled burning of char particles. Particularly for large particles of radii 0.1 to 1.25 mm at temperature 1200 K, the diffusion resistance seems to be significant compared to kinetic resistance. There are various reaction schemes adopted for char burning under diffusion control. It was shown by Annamalai (1983) that the classical two film model by Avakians and Davidson (1975) and as well as the heterogeneous direct combustion of C to CO yield identical burning rates when compared under identical conditions irrespective of whether the char burns in quiescent or flowing atmosphere as long as buoyancy induced momentum around the particle is insignificant which is the typical case in fluidized bed combustors. For diffusion controlled burning,

\[
\begin{align*}
\dot{a}_{ch} &= 2 \sqrt{\gamma} \ln(1+\eta)/\eta \frac{a_{ch}}{C_h} \\
&= 2 \theta \sqrt{\gamma} \eta / \eta \frac{a_{ch}}{C_h} \\
\end{align*}
\]

(51)

where

\[
\begin{align*}
\theta &= \gamma_{o} / \gamma_{s}, \gamma_c \\
Sh &= (2 + 0.6 \Re 0.5 \Sc 1/3) \frac{1}{f_{ch}} \\
\end{align*}
\]

(52)

(53)

(54)

Kinetic Controlled Burning

If \( m = -2 \) and \( n = 1 \), explicit results for kinetics controlled burning can be obtained from eq. (44).

\[
\frac{f_{ch}}{1 - (24/330)} = \frac{L}{1 + (24/330)}
\]

(55)

The curve CD in Figure 5 shows the results for burnt char fraction under kinetics controlled. Note that as \( n \to 0 \), the initial size \( R_e \to 1 \), \( W \) approaches zero and \( f_{ch} \) to unity. The parameters \( D_0 \) and \( a_2 \) have to be determined from the kinetics of heterogeneous oxidation. For a first order heterogeneous oxidation following the reaction

\[
C + 1/2O_2 \rightarrow CO
\]

\[
\frac{\dot{F}_{ch}}{1 - (24/330)} = \frac{L}{1 + (24/330)}
\]

(56)

Thus

\[
\begin{align*}
\dot{a}_{ch} &= - \frac{D}{1 - (24/330)} \frac{L}{1 + (24/330)} \\
&= \frac{D}{1 - (24/330)} \frac{L}{1 + (24/330)} \\
\end{align*}
\]

(57)

Figure 5 can be readily used for any FBC provided \( D_0 \) can be calculated or estimated from geometric and operating parameters of FBC. As particle loss rate is increased or as the reactivity is decreased the burnt char fraction always decreases.
It should be noted that the results shown in Figure 2 to 5 are valid for any FBC fired with arbitrary solid fuels irrespective of whether homogeneous oxidation (CO + 1/2O₂ ⎯→ CO₂ or V + O₂ ⎯→ CO₂ + H₂O) takes place or not. Homogeneous oxidation affects the value of V through the term V_o₂.

Correlation of Char Combustion Efficiency With Damköhler Number V

Figure 6 shows the comparison between the theoretical data and the several experimental data from Gibbs (1978) and McLaren (1969) both for the diffusion and kinetics controlled burning. The burn char fraction always increases with the increase in Damköhler number V, because high Damköhler number V implies the low attrition rate and longer particle residence time compared to the reaction time of the particles. The experimental data correlates very well with the Damköhler number V. This figure can be used to predict the burn char fraction if the Damköhler number V which contains the various important parameters of fluidized bed combusters such as air velocity, radius of particles, weight of the bed and the fluidized bed geometry is known.

THERMAL PERFORMANCE OF FBC

For predicting the thermal performance of bed at different levels, the bed temperature and combustion efficiencies must be plotted as a function of load (fuel feed rate). For the quantitative results, constant air flow rate and varying solid fuel firing rate were used. Such an approach of constant air flow will justify assumptions of i) perfectly stirred emulsion (phase), ii) constant overall heat transfer coefficient between the bed and heat exchangers and iii) constant elutriation coefficient. It is assumed that λ₂ = 1.0. The data used for calculation are shown in Table 1.

Ignition and Extinction

Consider the solid coal particles fired into a FBC. Coal devolatilizes and burns. While the volatiles partially burn in the FBC, the solid particles are almost completely burnt either because of longer residence time or because of capture of the particles in the hot cyclone and return back to the FBC. Thus first it is useful to study the behavior of FBC when compatible to 1. Results were also obtained when f = 1 and they will be published in a later paper. Figure 7 shows the results for bed temperature vs. input coal mass fraction for a coal of 40% volatile matter and 60% char and for a fixed air flow rate. Thus as the coal feed is decreased with same air flow, sensible heat loss is same for decreased heat generation and hence the bed temperature decreases. For Yₜ < 0.0535 there are three solutions. The curve ABC is the result of almost complete combustion of volatiles while the curve DEF is the result of negligible combustion of volatiles. The curve CD is the unstable part and is of no practical interest. If input coal mass fraction exceeds 0.0535, volatiles are ignited in the bed, the FBC operates in the regime EF for Yₜ > 0.0535. If Yₜ < 0.0535, volatiles can no longer burn in the bed, the FBC operates in the regime EF provided that the chars are reactive enough at these temperatures to completely burn. Thus FBC, when operating in circulating mode, where the elutriated hot char particles are returned back to the bed, can be operated at low bed temperature with free board volatiles burning if char combustion efficiency is maintained at a constant value. (Though not showing the curve EF can be extended up to Yₜ = 0.02 at which the char in the feed is enough to supply sensible heat less at T > 600 K.) This low temperature mode of operation is quite advantageous if the problems of low melting ash agglomeration is a problem in FBC as faced in operation of biomass fired FBC. For 0.0535 < Yₜ < 0.0535, the FBC can have thermal excursions between the regimes EF and CB if sufficient thermal disturbance is introduced in the bed.

So far the discussions concerned only with results of FBC temperature with assumption of complete combustion of char. If finite kinetics of combustion of char to CO₂ is assumed, then there appears to be only one possible operational range A'B'C' (dotted line in Figure 7). The middle range CD' is unstable part and is of no interest. The lower range very near to ambient temperature is not shown and is not relevant. As seen in Figure 7, volatiles combustion is negligible for T < 800 K and hence in the region F'D' in Figure 7, only char is burning. Figure 8 shows the burn char and volatiles fraction with bed temperature. It should be observed that the burn char fraction decreases slowely with increase in temperature compared to volatiles and compared to single particle burning rate. The reason is that as the temperature decreases, the weight of the carbon in the bed increases which increases number of reacting particles resulting in more surface area available for oxidation at lower temperature. As such that for those fuels with low volatile matter multiple steady states may not occur even though a single particle can exhibit such behavior!

In practice operation of FBC, it is of interest to know the turn down capability of the bed. Assuming that the bed can be operated from the stoichiometric condition Yₜp = 0.091, then the minimum allowable (Yₜp) is 0.050, for the case of kinetics controlled burning of char. Thus average turn down ratio (TD) is about 1.67 for a coal of 40% volatile matter and probably TD could be improved to 2.00 with return to solid char particles back to the FBC. These values are far below TD which can be achieved in suspension fired boilers.

Comparison Of Present Model With Other Models

The present theory was developed with a view to obtain a few dimensionless groups governing the thermal performance of the bed. Hence simplifying assumptions such as frozen bubble phase, uniform reaction mechanism for all particles in the bed etc. were made in obtaining the results. Thus it is
necessary to compare the present results with results from rigorous models. Figure 9 compares the results for combustion efficiency (defined as a ratio of heat release/fuel firing rate heating value) with results obtained by Georgakis et al. (1981). The solid line shows the results obtained with present theory, while the dashed line shows the results of Georgakis. It should be noted that while the present theory uses $C + \frac{1}{2}O_{2} \rightarrow CO_{2}$, $CO + \frac{1}{2}O_{2} \rightarrow CO_{2}$ (instantaneous) and $V + O_{2} \rightarrow CO + H_{2}O$ (kinetic constants same as $CO$ oxidation kinetics of Georgakis (1981)), the theory by Georgakis uses $V + O_{2} \rightarrow CO + H_{2}O$ (instantaneous), $C + \frac{1}{2}O_{2} \rightarrow CO_{2}$, and $CO + \frac{1}{2}O_{2} \rightarrow CO_{2}$. The agreement was excellent. However, when we used the reaction mechanism $C + \frac{1}{2}O_{2} \rightarrow CO_{2}$, $V + O_{2} \rightarrow CO + H_{2}O$, and $CO + \frac{1}{2}O_{2} \rightarrow CO_{2}$, the present results (dotted line) are lower by about 3.0 to 7.5 percent compared to the dashed line. The explanation for this difference is that we assumed that the volatiles burn completely to $CO_{2}$ liberating less heat. Georgakis et al. (1981) assumed that the volatiles burn completely to $CO_{2}$ liberating more heat. It is also observed that the qualitative trends are similar.

SUMMARY

1. A simplified theory is presented in the paper which shows that most of the experimental data on bed char weight (char) combustion efficiencies can be correlated with a single nondimensional group called as Damköhler number $D$.
2. The correlation was verified with experimental data reported elsewhere.
3. The present theoretical results (two nonlinear algebraic equations) were also verified both qualitatively and quantitatively with results obtained from rigorous theory (differential and integro-differential equations).
4. It is shown that the bed char weight can be used to interpret the global reaction mechanism of burning of char particles in the bed.
5. Thermal performance indicates that there exist ignition and extinction regimes in the bed due to gas phase combustion.
6. While multiple solution exists for the burning rate of a single char particle such solutions may not exist for the total burning rate of all char particles since the number of particles at any time in the bed increases if reaction rate decreases and vice versa and the total burning rate, vs. the temperature may no longer be an exponential curve.

ACKNOWLEDGMENT

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W  Weight of char in the bed
Y  Species mass fraction
Y_v,i  Volatile mass fraction at inlet
2  Equation (7)

Greek Symbols

\[ \alpha \]  Burning rate constant
\[ \beta \]  Burning rate constant under diffusion controlled burning
\[ \beta_c \]  Burning rate constant under kinetics controlled burning
\[ \theta \]  7/T
\[ \gamma \]  Stoichiometric coefficient for volatiles
\[ \gamma_c \]  Stoichiometric coefficient for char
\[ \phi \]  Eq. (35)
\[ s \]  Density
\[ t \]  Eq. (34)

Subscript

b  bubble phase
bm  bed material
c  char
e  emulsion phase
g  gas
H  at freeboard
i  free stream condition
mf  minimum fluidization
o  oxygen
v  volatiles
w  heat exchanger wall

REFERENCE

Annamalai, K., (1980), 'Extinction Studies of Fluidized Bed Combustors,' Presented at Central States Section of the Combustion Institute, University of Kentucky, Lexington, CSS/CI 83-12


Kapp, G. S., and Harvey, W. T., Jr., (1979), Fluidized Bed Combustion, Where We Are Today, Kentucky Industrial Coal Conference, Kentucky.


Table 1. DATA

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FIGURE 1. a. SCHEMATIC DIAGRAM OF FLUIDIZED BED COMBUSTOR.

FIGURE 1. b. BUBBLE AND EMULSION PHASE.
FIGURE 2. GENERALIZED RESULTS FOR BED WEIGHT AND BURNED CHAR FRACTION WITH INVERSE OF DAMKOHLER NUMBER $V$.

FIGURE 3. DIMENSIONLESS CHAR BED WEIGHT VS. INVERSE OF DAMKOHLER NUMBER $V$ FOR KINETICS AND DIFFUSION CONTROLLED COMBUSTION.

FIGURE 4. WEIGHT VS. EXIT OXYGEN %. A COMPARISON BETWEEN EXPERIMENT AND CHM THEORY.

FIGURE 5. CHAR BURNED FRACTION VS. INVERSE OF DAMKOHLER NUMBER $V$. 
Figure 7. Ignition and extinction regimes.
Burnt char fraction = 1 (assumed)
Actual kinetics controlled burnt char.