Proceedings of the
ASME ADVANCED
ENERGY SYSTEMS DIVISION

• DIRECT THERMAL POWER CONVERSION
  AND THERMAL MANAGEMENT
• THERMODYNAMICS AND THE DESIGN, ANALYSIS,
  AND IMPROVEMENT OF ENERGY SYSTEMS
• HEAT PUMP AND REFRIGERATION SYSTEMS, DESIGN,
  ANALYSIS AND APPLICATION

presented at
THE 1997 ASME INTERNATIONAL MECHANICAL ENGINEERING CONGRESS AND EXPOSITION
NOVEMBER 16–21, 1997
DALLAS, TEXAS

sponsored by
THE ADVANCED ENERGY SYSTEMS DIVISION, ASME

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THERMODYNAMICS AND THE DESIGN, ANALYSIS, AND IMPROVEMENT OF ENERGY SYSTEMS

Introduction

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It is my pleasure to present this proceedings of the 13th Symposium on Thermodynamics, and the Design, Analysis, and Improvement of Energy Systems, a symposium sponsored by the System Analysis Technical Committee of the ASME Advanced Energy Systems Division. About one year ago, when Professor Krane so cordially invited me to become the Chairman of the 13th Symposium, I considered the skills needed to devise the technical sessions, to recruit the chairs and co-chairs, to coordinate the review process, to make sure all papers were handled fairly and timely, and to complete the task by August 19, 1997 (ASME deadline). I know now what is needed — in order of importance: time, patience, impartiality, and organizational skills. Obviously, a very efficient secretary would be a plus (I was my own secretary, so I learned what it takes).

A total of 51 abstracts were received for consideration, 25 of them accepted for publication and presentation after a careful review process. The accepted papers were divided into the following seven sessions: Fundamentals of Thermodynamics; Exergy in Industrial Processes; Energy in Industrial Processes; Thermoeconomics; Innovative Thermo-Processes; Unconventional Thermodynamics; and Thermo-Numerical Analysis. The symposium sessions are very well balanced with approximately 60 percent of the papers being of practical relevance and 40 percent being of a more fundamental importance. Thermoeconomics and exergy analysis continue to play an important role in the symposium. A special session on Unconventional Thermodynamics presents some unconstrained research dealing, for instance, with the analysis of human fever and with a new theory of natural optimization called “constructal.” The growth of computer use for simulating thermal processes is highlighted in the session Thermo-Numerical Analysis.

The technical sessions are followed by panel discussions on “Research Needs, Industry Trends, and Academic Partnership” on November 20, 1997 and “Thermoeconomics” on November 21. The panels are both provocative and industry related, providing an informal atmosphere for the exchange of ideas and interaction with the leaders in the field.

I want to extend my personal gratitude to Prof. James Chapman for his up-and-above support, and to the chairs and co-chairs (listed as co-editors) for their help with the review process, and especially those who returned my e-mails and telephone messages promptly, and those who respected the deadlines! Their efforts, as well as the efforts of the reviewers (listed next) and the authors, are the backbone of the symposium.

Finally, I appreciate the confidence in my scholarship and aptitude for organizing the symposium. I feel good for having had the opportunity to collaborate as an ASME volunteer — with no need for brownies!
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A THERMODYNAMIC ANALYSIS OF FEVER IN CHEMOTRAPHES

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ABSTRACT
Temperature regulation of chemotraphs (animals and humans) is one of the most sophisticated processes in nature. The basal condition is characterized as the absolute minimum energy state during consciousness. The conditions necessary are: no food for twelve hours, restful sleep, complete rest when awake, no excitement, and an ambient temperature of 20-27°C. The body’s metabolism of chemotraphs is constantly producing excess heat, even at rest that must be removed in order to maintain thermal equilibrium. Under normal conditions the heat produced by the biochemical reactions within the body must be the same as the heat loss. A first law analysis is conducted in determining the net rate of heat-loss while the body is at rest. The results are then compared with data obtained by physiologists for humans and checks to within 5%. The analysis is extended to predicting the occurrence of fever. Dimensional results are then presented for the body temperature vs time. Fever can occur either through increased slow oxidation/combustion (metabolic oxidation) of a mix of carbohydrates and fats, or by reduced heat-loss from the skin due to reduced blood flow (which causes shivers). If all the energy released during oxidation/combustion results in direct heat production then it is possible to predict I) temperature vs time, and ii) the initial metabolic rate from nasal exhaust measurements during initial periods of fever. Good agreement has been obtained between model results and experimental data on cats during heat-up periods.

NOMENCLATURE
English Symbols
- \( h \) - total enthalpy of species k (kJ/kmole)
- \( h_{a,k} \) - enthalpy of formation/chemical enthalpy of species k (kJ/kmole)
- \( h_{T,k} \) - thermal enthalpy of species k (kJ/kmole)
- \( H' \) - heat
- \( M \) - metabolic activity
- \( m \) - mass flow rate of breathing air (kg/s)
- \( m_b \) - mass of the body (kg)
- \( N \) - gas mole flow
- \( P \) - pressure
- \( P_{a,k} \) - partial pressure of species k (bar)
- \( Q \) - total heat transfer to the body (J)
- \( T \) - temperature (°C)
- \( t \) - time (sec)
- \( t_c \) - characteristic time
- \( U \) - internal energy
- \( V \) - dry volume flow rate (m³/s)
- \( V_{O_{2,LU}} \) - specific oxygen utilization rate (m³/kg/s)
- \( W \) - total work done by the body (J)
- \( Y_k \) - mole fraction of species k

Greek Symbols
- \( \alpha \) - see Equation (15b)
- \( \beta \) - heat generation ratio
- \( \varepsilon \) - emissivity
- \( \theta \) - temperature normalization ratio
- \( \nu \) - stoichiometric oxygen per unit fuel (kg of O₂/kg of fuel)
- \( \tau \) - \( \psi_c \)

Subscripts
- a - air
- b - burned
- B - body

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c - characteristic
cv - control volume
e - exhaust
ext - external
F - fuel
f - fuel
gen - generated
I - inhaled
int - internal
m - moisture
n - normal
rad - radiation
s - skin
u - utilized
∞ - ambient

Acronyms
ADP - adenosine diphosphate
ATP - adenosine triphosphate
BMR - basic metabolic rate
CH - carbohydrate
EP - endogenous pyrogen
Hb - hemoglobin
HHV - higher heating value
HV - heating value of burned fuel kJ/kg
LHV - lower heating value
OHB - oxy-Hemoglobin
PGE - prostaglandin of the E series
RER - respiratory exchange ratio
TV - tidal volume

INTRODUCTION
Metabolism is defined as a sum of all processes (chemical and physical) resulting in energy conversion. Our metabolisms are constantly producing excess heat within our bodies that must be removed in order to maintain thermal equilibrium. Temperature regulation of the human body is one of the most sophisticated processes in nature. Although the environment and daily activity might give people a cause to believe that body temperature varies significantly throughout the day, it has been proven that temperature rarely exceeds ±1°C from the average basal core temperature of 37°C [Du Bois]. It should also be noted that there is no single core temperature. The basal condition is characterized as a persons absolute minimum energy state during consciousness. The basal conditions are: no food for twelve hours, restful sleep, complete rest when awake, no excitement, and an ambient temperature of 20-27°C. Fortunately, our bodies have the ability to dispel heat through the flow of blood from the body core to the outermost layers of skin. The heat is conducted to the surface of the skin, then convection and radiation transfer the heat away from the body. Heat is also removed by perspiration and evaporation from the mouth and nose.

Fever is defined as body temperature above 98.6°F (oral) and 99.8°F (rectal). The temperature in a local region depends upon the metabolic rate of the surrounding tissues, source and magnitude of blood flow and any local heat-loss. There has been much study and research in physiology aimed at understanding fever and the effects of exercise on the body. Body temperature increases with exercise and heat stress, and rectal temperatures as high as 40-42°C have been recorded during exercise [Hubbard, and Armstrong]. About 70% of the basal metabolic rate, BMR, is produced in the internal organs for resting humans, while 90% is produced by skeletal muscles (hence higher skeletal temperatures) during exercise. Thus, the cells adjacent to skeletal muscles will be at higher temperatures during exercise. The release of heat is coupled to the oxidation of "fuel" the products of which are expelled, via blood and the lungs, to the atmosphere through nasal exhaust. Seemingly no work has been done on the relationship between the conservation of energy and nasal exhaust composition during fever. Since core temperature depends upon the balance between heat gain and heat-loss, a model should be established to determine core temperature under normal conditions and the onset of fever conditions.

The overall objective of the present work is to give the reader a fundamental understanding of the origin of fever in terms of the thermodynamics conservation of energy principles. In order to achieve the objective, a model was presented for establishing the link between respiration gases and fever. Finally, the model results were verified with existing measurements on metabolic rates, under normal conditions. The model was then extended to present temperature rise vs time for the onset of fever conditions and compared with experimental data.

LITERATURE REVIEW
Chemotrophs are species which obtain energy from the oxidation of food; e.g., humans and animals. Phototrophs are species which obtain energy by trapping light energy [Stryer]. There are three stages in extraction of energy for the chemotrophs: food broken down into smaller molecules, smaller molecules degrading to simple units, and finally simple units oxidized to CO₂ and H₂O in tissues. The oxygen reaches the tissues via the following process. Since O₂ in the lungs cannot directly dissolve in blood it then attaches to hemoglobin (Hb, 150 g/L of blood) and becomes oxy-Hemoglobin (OHB) in a few milliseconds.

Energy produced by the oxidation process in tissues is carried by ATP (Adenosine Triphosphate) to different parts of the body where it is hydrolyzed, releasing energy. ATP is formed from ADP (Adenosine Diphosphate) when fuel
molecules are oxidized. Typically, the ATP molecules which serve as the energy carriers absorb 27% of the energy released during oxidation process. A fraction of the energy is used to rebuild decayed cells and to perform respiratory functions. Typical fuels for oxidation reactions are fats and carbohydrates (CH) [Berne and Levy]. Global reactions are given as

$$\text{(CH}_2\text{O)}_m + m \text{ O}_2 - m \text{ CO}_2 + m \text{ H}_2\text{O} \quad (1a)$$

An example of CH is glucose, where m equals six. The enthalpy of combustion for glucose is -2.808 MJ/kmole. The reaction of fat (Palmitic acid) is represented by

$$\text{C}_{16} \text{H}_{32} \text{COOH} + 23 \text{ O}_2 - 16 \text{ CO}_2 + 16 \text{ H}_2\text{O} \quad (1b)$$

It is noted that the H to C atom ratio is two for both fat and CH. Normally, during oxidation, only 25% of the O2 from OHb is exchanged with the tissues. thus, there is almost a 300% excess of O2 in the blood. Under the exercise conditions mentioned above, almost 50% is exchanged (i.e., 100% excess). A typical product of the reduction reaction is ammonia, the major component of urine:

$$\text{N}_2 + 3 \text{ H}_2 \rightarrow 2 \text{ NH}_3 \quad (2)$$

It is not necessary to have an intake of food to start this metabolic process because it is always occurring in every living being. At rest, the work is mostly internal and the oxygen consumption by the body is at its least, typically, around 260 mL/min or 88 W for a person of surface area 1.8 m². This translates into a heat release of 20,500 kJ/m of oxygen consumed.

The hypothalamus is located in the brain and acts as a thermostat for the body; it is responsible for maintaining the body temperature at 37°C. When an infection enters the body, leukocytes, or white blood cells, are dispatched to surround and contain the infection. These leukocytes release a chemical substance called endogenous pyrogen (EP), which is a small protein. These pyrogens elevate the regulated set-point temperature of the body through an increased concentration of prostaglandins of the E series (PGE). When PGE is released into the blood and reaches the brain, the temperature set-point is increased to a new value [Houdas]. However, it will be assumed that this is not the case, and EP is an adequate producer of PGE.

The body's response to this new temperature setting is a desire to increase heat production via slow oxidation combustion process of carbohydrates and/or fats, and to decrease the heat loss via reduced blood flow in order to achieve a new thermal equilibrium [Mackowiak]. The priority of all metabolic oxidation reactions is now to create heat. Typically, the average skin temperature is 33.5°C, under normal conditions, but with a large decrease in blood flow this temperature will fall to near ambient values. Inherently, this condition allows negligible heat transfer from the body to the environment. A new metabolic rate is established that is higher the normal rate because it is temperature dependent. In fact, it has been estimated that the metabolic rate increases by 13% for each 1°C rise in temperature, which yields an activation energy of 98,000 kJ/mkole, and 50% above normal at 40.6°C [Callaham]. Mackowiak reports of twice the metabolic rate for every 10°C rise in temperature with activation energy of 59,000 kJ/kmole.

There is a suppression of the sense of hunger which causes a lack of appetite during fever, therefore the body must resort to using its own potential/chemical energy stores in order to fuel metabolic functions. A decline in the concentration of PGE is a signal that the infection has been defeated and the set-point returns to 37°C. The brain senses that the heat loss must increase and the heat production must decrease, thus slowing the combustion processes. The blood flow to the skin is increased and typically, there is an enormous amount of sweating, resulting in an increase in the evaporation process for humans; animals do not sweat. Eventually, thermal equilibrium is returned to normal and the metabolic rate goes back to its normal state.

Research has been done to relate body temperature to oxygen consumption during exercise. Empirical correlations suggest that during exercise:

$$T = 37.25 - 0.00264 \times (\% \text{ of VO}_2 \text{ max}) + 0.00037 \times (\% \text{ VO}_2 \text{ max})^2$$

where the temperature is taken in °C [Davies]. Apparently, no relationship has been found between oxygen utilization and body temperature during fever.

**THERMODYNAMIC ANALYSIS**

Prior to presenting the analysis, a brief discussion is given on energy balance and the respiration process. Food intake provides energy input as shown in Fig. 1 [Berne]. Energy output is used within the body for 1) mechanical work (respiration, blood circulation, cell movement), ii) synthetic reactions (fuel storage, tissue building), iii) membrane transport, iv) signal generation (electrical, mechanical and chemical), v) direct heat production, and vi) detoxification (urea formation, oxidation and reduction reactions). All these irreversible processes that occur within the body result in the dissipation of heat. It is noted that only a fraction of energy input is used for "direct" heat production.

The main goal of respiration is the exchange of gases used to supply oxygen, necessary for oxidation, and to remove the by-products of combustion. In thermodynamics, one is interested in all of the gases that enter and exit the lungs [Guyton]. One is also interested in the fuel, which may be a mixture of carbohydrates and fats (see Eq. 1), that is involved in the oxidation process. The oxidation process releases CO₂
and H₂O throughout the body, eventually reaching the lungs. Table 1 lists the partial pressures of inhaled (atmospheric) and exhaled gases handled by the lungs [Du Bois 1948; Milsom 1988]. The partial pressures are the same as mole fractions at P = 1 bar.

Table 1. Partial Pressures of Respiratory Gases (P = 1 bar)

<table>
<thead>
<tr>
<th></th>
<th>Inhaled Gas</th>
<th>Exhaled Gas</th>
</tr>
</thead>
<tbody>
<tr>
<td>N₂</td>
<td>0.7862</td>
<td>0.745</td>
</tr>
<tr>
<td>O₂</td>
<td>0.2084</td>
<td>0.157</td>
</tr>
<tr>
<td>CO₂</td>
<td>0.0004</td>
<td>0.036</td>
</tr>
<tr>
<td>H₂O</td>
<td>0.005</td>
<td>0.062</td>
</tr>
</tbody>
</table>

Fuel Composition

In order to determine the overall composition of the fuel mixture, an assumption is made that fuel mixture can be represented by a generic fuel, (CH₄O)ₙ, which produces CO₂ and H₂O when oxidized with inhaled air. Eq. (1) is then modified for the generic fuel with air as the oxygen carrier. The overall combustion equation is given as:

\[
\text{CH}_4\text{O}_y (s) + a \text{O}_2 + b \text{N}_2 + c \text{H}_2\text{O} + d \text{CO}_2 + e \text{H}_2\text{O} + f \text{N}_2 \rightarrow \text{CO}_2 + \text{H}_2\text{O}
\]

There are ten unknowns: x, y, a, b, c, d, e, f, g, and h. From an atom balance of C, H, O, and N, four equations are obtained. The percent of O₂, CO₂, and H₂O is known from the inhaled air composition, and the percent of O₂, CO₂, and H₂O is known from the exhaled gas composition; thus six additional equations are generated. Hence all the unknowns can be determined.

Mole and Volume Flow Rates

Knowing the ratio of product moles to gaseous reactant moles from Eq. (1) and the volume flow rates of inhaled air, one can estimate the moles of exhaled gases. Thus,

\[
N_k = c + d + e + f
\]

and

\[
N_k = a + b + g + h
\]

Since the volume displacement rates, of the human engine, are the same the ideal gas law yields,

\[
P_e/P_i = (N_e/V_e)/(N_i/V_i)
\]

It is seen that the pressure in the lungs at the time of exhalation is slightly higher than the pressure of the inhaled gases. The mole ratios of inhaled air to exhaled gases can also be expressed in terms of standard volume. N₂ is conserved under steady state; using the equal molar flow rates of N₂ in the intake and exhaust yields,

\[
V_{e}/V_{i} = Y_{N_2,e}/Y_{N_2,i}
\]

(4d)

Hence,

\[
V_{O_2,h} - V_{O_2,e} = Y_{k,i} - Y_{k,e}
\]

(4e)

where \( Y_k \) represents the mole fraction of species k on a dry basis and the variable V represents the dry volume flow rate. Hence, knowing the intake volume flow rates, and the N₂ and O₂ concentrations in the intake and exhaust, the oxygen used in the combustion process can be determined. Typically, the values \( V_{O_2,i} \) are tabulated in medical and kinesiology literature under normal exercise, and fever conditions.

Energy Analysis

The First Law of Thermodynamics requires an energy balance. The First Law can be presented for an open system with the human body as the control volume (Fig. 2). The gas enters and leaves the body via the nose. In addition, moisture leaves through the skin.

Energy stored in the body = heat gained from the environment + enthalpy-in through the nasal passages + energy-in via food intake - enthalpy-out through the nasal passages - enthalpy-out through excretions - enthalpy-out via perspiration - Enthalpy-out through dead tissues - work leaving the system.

In mathematical form, for a person at rest and between eating and excretion periods,

\[
dU/dt = \dot{Q} + \sum N_{k,i} \dot{h}_{k,i} - \sum N_{k,e} h_{k,e} - \sum N_{m} h_{m}
\]

(5a)

where total enthalpy of species k is given as

\[
h_k = h_{k,0} + h_k
\]

lumping \( \dot{Q} = N_{m,i} h_{m} \) as \( \dot{Q} \)

\[
dU_{e}/dt = \dot{Q} + \sum N_{k,i} \dot{h}_{k,i} - \sum N_{k,e} h_{k,e}
\]

(5b)

Even when the body is under resting conditions and basal temperature, metabolic reactions continue to occur, decreasing the mass; therefore, mass slightly decreases and energy level U changes. It will also be assumed that the analysis is conducted in between eating periods, the chemical heat source is dominated by the oxidation process, the mass decrease is essentially due to oxidation of generic fuels, and the rest of the mass is inert. The presence of enzymes and catalysts affects only the rate of reaction but not the final
product [Atkins]. Therefore, the enzyme and catalyst mass does not change under steady state. Eq. (5b) can be reduced to the following form by accounting for the decrease in mass and using the definition for total enthalpy,

\[ m_{A_{BH}} \frac{dT}{dt} = \frac{d}{dt} \left( m_{A_{BH}} \cdot \text{HV}_F \right) + m_{c_{A_{BH}}} \left( T_s - T_e \right) + Q \]  

(5c)

where the lower heating value LHV is defined as

\[ \text{LHV}_F = \left[ h_F - u_{CO}_2 \cdot h_{CO}_2 - u_{H_2O} \cdot h_{H_2O}(g) \right] \]  

(5d)

and \( c_{A_{BH}} \) is the specific heat of the body (= 3.5 kJ/kg K).

**Case (a): Normal Conditions**

At normal steady state conditions, Eq. (5c) yields,

\[ \dot{Q}_{gen} = \dot{Q}_a + \dot{R}_p \cdot e_a \left( T_s - T_e \right) \]  

(6a)

where

\[ \dot{Q}_{gen} = \dot{m}_{F_{BH}} \cdot \text{HV}_F \]  

(6b)

and \( \dot{Q}_a \) is the heat loss through skin under normal conditions. Eq. (6a) implies that heat generation, which occurs predominantly by metabolism, is equal to the heat loss rate through the skin and the dry-loss (non-evaporative). Since the mole flows in Eq. (6a) are assumed to occur only through the nasal passages, \( \dot{Q}_a \) is interpreted as the global heat-loss due to conduction, convection, radiation, and perspiration through the skin's surface. Typically, the dry-loss is very small (2%) compared to the heat-loss through the skin (98%), therefore from Eq. (6a),

\[ \dot{Q}_a = \dot{Q}_{gen} = \dot{m}_{F_{BH}} \cdot \text{HV}_F \]  

(6c)

Since the A:F ratio and the breathing rate are known, \( \dot{m}_{F_{BH}} \) is also known. By knowing the \( \text{HV}_F \) and the breathing rate, \( \dot{Q}_{gen} \) can be calculated from Eq. (6b). Then \( \dot{Q}_a \) can be determined from Eq. (6c). Therefore, the Boile equation is recommended for estimating the higher heating values (HHV) of fuels. This equation has been tested for 16 biomass fuels and 67 oil fuels, including alcohols. Recently it has been used for 47 types of plants and 6 different feedlot manures [Annamalai et al]:

\[ \text{HHV} (kJ \text{ per kg of fuel}) = 35160 \, C_m + 116 \, 225 \, H_m - 11090 \, O_m + 6280 \, N_m + 10465 \, S_m \]  

(7a)

where \( C_m, H_m, \) etc., are in kgs of element per kg of fuel. One can arrive at a formula for stoichiometric oxygen in terms of elemental composition, estimate the heating value using Eq. (7a), and derive at the following formula:

\[ \text{HV}_O_2 \]  

(7b)

where \( C, H, \ldots \) etc. are carbon, hydrogen, \ldots atoms in the fuel Thus, Eq. (6c) becomes

\[ \dot{Q}_a = \dot{Q}_{gen} = \dot{m}_{O_2} \cdot \text{HV}_O_2 \]  

(8a)

where \( \dot{m}_{O_2} = \dot{m}_{F_{BH}} \cdot v \) where \( v \) is the stoichiometric oxygen to fuel mass ratio. Rewriting Eq. (8a) in terms of volume of oxygen used by the body and then dividing by the mass of the body \( \dot{m}_b \),

\[ \frac{\dot{Q}_a}{\dot{m}_b} \cdot \dot{m}_b = \dot{V}_{O_2} \cdot \text{HV}_O_2 \]  

(8b)

where \( \dot{V}_{O_2} \) will be called the specific oxygen utilization rate \((m^3/s)\).

**Case (b): Fever**

Heat-loss from the body occurs by convection, radiation, and evaporation of moisture through the skin, accounting for 43%, 49% and 8%, respectively [Proppe]. Considering convection, radiation, and perspiration under fever conditions,

\[ \dot{Q} = h \cdot A_s \cdot (T_s - T_e) + \sigma \cdot A_s \cdot (T_s^4 - T_e^4) + h \cdot h \cdot A_s \cdot (P_{H_2O}^s - P_{H_2O}) \]  

(9a)

where \( T_s \) is the skin temperature. \( h \) is the bulk convective heat transfer coefficient, and \( h_m \) is the heat loss coefficient by mass transfer. Since \( T_s \) is very small compared to \( T_e \), the second term can be expanded using a binomial expansion. For \( H_2O \), the partial pressures can be replaced by saturation pressures under profuse sweating conditions as

\[ P_{H_2O} = P_{H_2O}^s \]  

(9b)

for 25°C<40°C. Therefore, Eq. (9a) is written as

\[ Q = h \cdot A_s \cdot (T_s - T_e) \]  

(9b)

where

\[ h = h_e + h_{rad} + h_m \]  

(9c)

\[ h_{rad} = \varepsilon \sigma \cdot T_s^4 \]  

(9d)

\[ h_e = 0.00276 \cdot h_m \cdot h_{ig} \]  

(9e)

where \( h_{ig} \) is enthalpy of vaporization. Typically, the \( \varepsilon = 0.98 \) for skin and it decreases to 0.95 with clothes. It has been established, empirically, that the mass transfer coefficient \( h_m \) is 1.672 \( h_e \) \((kW/m^2 \cdot \text{bar})\), known as the Lewis relation [Rapp]. The "h" should be interpreted as the global heat
transfer coefficient. The total heat-loss is proportional to the difference between the body temperature and the ambient temperature, therefore, Eq. (5c) can now be reduced as,

\[ m_b \cdot c_{p,b} \cdot \frac{dT}{dt} = -h \cdot A_s \cdot \left| T - T_s \right| + \dot{Q}_{gen} - m_p \cdot c_{p,A} \cdot \left| T - T_J \right| \]  

(10a)

As the blood flow is reduced, \( T_J \) decreases reducing the heat-loss. Letting

\[ F_b = \frac{(T_f - T_s)(T - T_n)}{(T_n - T_s)} \]  

(10b)

with \( Q_h = h \cdot A_s \cdot (T_n - T_s) \), Eq. (10a) can be transformed to,

\[ m_b \cdot c_{p,b} \cdot \frac{dT}{dt} = -Q_h \cdot F_b \cdot \left| T - T_n \right| \left| T_n - T_s \right| + \dot{Q}_{gen} - m_p \cdot c_{p,A} \cdot \left| T - T_J \right| \]  

(10c)

The factor \( F_b \) has been introduced in order to account for any reduction in heat-loss with reduced blood flow. For \( F_b = 1 \), the blood flow is the same as under normal conditions and \( T_J = T_n \).

It is assumed that the skin temperature is uniform throughout the body. The exhaled air is assumed to be the same as the body’s core temperature. Equation (10a) can be integrated and the solution for \( T(t) \) can be obtained. The following normalization can now be introduced:

\[ \tau = \frac{t}{T_n} \]  

(11a)

\[ \theta = \frac{(T - T_n)}{(T_n - T_s)} \]  

(11b)

\[ \beta = \text{heat generation under fever/heat generation under normal conditions} \]  

(11c)

In view of Eq. (8b),

\[ \beta = \frac{v_{O_2,uf}}{v_{O_2,un}} \]  

(12)

where \( v_{O_2,uf} \) is the specific oxygen used under fever. It should be noted that \( \beta \) is not a constant because the rate of oxygen consumption varies with time during fever.

\[ t_c = \frac{c_{p,B} \cdot \left| T_n - T_s \right|}{k_{O_2,un} \cdot H V_{O_2}} \]  

(13)

The time \( t_c \) is interpreted as the characteristic time to heat or cool the body.

\[ \gamma = \frac{m_o \cdot c_{p,A} \cdot \left| T - T_J \right| \cdot \dot{Q}_{gen,n}}{\dot{Q}_{gen,un}} \]  

(14)

Eq. (10) then becomes

\[ \frac{d \theta}{dt} = -\alpha \theta + \beta \]  

(15a)

where

\[ \alpha = F_b + \gamma (F_b - 1) \]  

(15b)

Typically, \( \gamma = 0.02 \) therefore, \( \alpha = F_b \). Using normalization procedure with normal metabolic rates, one can generalize the equation for all chemotrophs. For e.g. The cats may have thick hair throughout the body which acts as radiator fins cooling the body with increased heat loss per unit surface area of skin while humans may have less hair with decreased heat loss per unit area. Eq. (15a) is applicable for both the species.

**RESULTS AND DISCUSSION**

**Fuel Composition**

Using the data given in Table 1, and the procedure outlined under thermodynamic analysis, the unknown constants in Eq. (3) can be determined. Thus, Eq. (3) becomes:

\[ \text{CH}_2\text{O}_2 + 5.544 \text{ O}_2 + 20.915 \text{N}_2 + 0.0106 \text{ CO}_2 + 0.133 \text{ H}_2\text{O} \]

\[-1.0106 \text{ CO}_2 + 1.741 \text{ H}_2\text{O} + 4.408 \text{ O}_2 + 20.915 \text{ N}_2 \]

(16)

It is apparent that the H to C ratio is different from the value of Eq. (1a) or (1b), as presented for glucose and palmitic fat, since a fuel mixture may be involved. While the stoichiometric oxygen to fuel mole ratio is 1.14, the actual \( \text{O}_2 \) to fuel ratio is much higher, which indicates that most of the \( \text{O}_2 \) is returned unused through the exhaust. Excess air is computed as 385%; this value agrees with literature that only 25% of \( \text{O}_2 \) in the blood stream is used for oxidation which indicates a 400% excess of oxygen in blood. With Eq. (7a), the higher heating value is estimated as 15,372 while the lower heat value is determined to be 14,779 kJ/kg.

**Heat Loss Under Normal Conditions**

Typical respiration rate is on the order of 0.0001 m³/s of air [Guyton]. Using the data in Table 1, the ideal gas law, Eqs. (5) and (6), the Boie equation, and assuming that \( T_n = 26.85^\circ \text{C}, T_s = 37^\circ \text{C}, \) the \( Q_{gen} \) is estimated as 82 W, (called basal metabolic rate, BMR), under normal conditions for an average person at rest. Heat-loss through the skin \( Q_s \) is estimated as 80 W and dry loss is only 2 W! The \( \text{O}_2 \) consumption rate is given as 0.0001*0.2184/4.85 = 4.3 cc/s for an average person of 70 kgs which translates to 3.7 cc/kg/min. The reported experimental data for \( \text{O}_2 \) consumption for BMR is 2.8 to 3.6 cc/kg/min at rest [Moon et al].

Though work is required to pump blood, all work is converted into frictional work and the overall control volume analysis accounts for the internal muscle work, which is dissipated as heat. As a check on the current calculations, the energy consumption is determined per kg, per minute. Since
the gas composition is for an average person of 70 kg (150 lb.), the metabolic rate per kg per minute is given as 0.070 kJ/kg/min. It is reported that under light activity, ranging from sleeping to standing conditions, the metabolic rate is given as 0.065 to 0.111 kJ/kg/min [Banhide]. Furthermore, the heat generated per m² of oxygen consumed is determined to be 19,300 kJ/m² while physiologists determined the value as 20,500 kJ/m² of O₂ consumed, using thermal equivalent data. This number seems to be a constant for most of the fossil fuels [Annalamalai et al, 1994].

Fever: Temperature vs Time

a) Constant β

Integration of Eq. (15) yields

\[ \theta = \beta / \alpha - [\beta / \alpha - 1] \exp (- \tau / \alpha) \]  
(17a)

\[ \theta = \beta' / \alpha' - [\beta' / \alpha' - 1] \exp (- \tau') \]  
(17b)

where \( \beta' = \beta / \alpha \), and \( \tau = \tau / \alpha \)

The \( \theta \) vs \( \tau' \) curves are obtained with \( \beta' \) as a parameter. Figure 3 shows a series of curves for different heat generation ratios (\( \beta' \)) that may be used to determine the amount of time required to reach non-dimensional temperature. At \( \tau = \tau' = 0 \), Eq. (15a) yields \( \theta / \theta_{\text{prev}} = \beta' - 1 \); then by monitoring the temperature vs time, one can predict the initial metabolic rate \( \theta_{\text{prev}} \) from Eq. (11c). Particularly, different infections result in different metabolic rates; the parametric plots illustrate their effects on \( \theta \) vs \( \tau \).

Typically, a temperature rise occurs over a period of about 2 hours [Guyton]. The selection of \( \beta' = 1.75 \) results in a temperature rise of about 2.5°C over a 2 hour time period, which is typical for humans. Figure 4 shows the temperature vs time with \( \beta' = 1.75 \). This \( \beta' \) value could be attained either under increased metabolic rates of \( \beta = 1.75 \) at fixed \( \alpha = 1 \) (blood flow to the skin is the same as under normal conditions) or at the same normal metabolic rate \( \beta = 1 \) with reduced \( \alpha = 0.57 \) (\( \alpha < 1 \), i.e., with 57% of normal blood flow to the skin) which generates the feeling of chilliness.

b) Variable β

The above analysis assumes \( \beta \) is a constant. However, as previously stated, \( \beta \) varies with time. Then integrating Eq. (15a) for a short period of time

\[ \theta = \theta_{\text{prev}} + [\beta(\tau) / \alpha(\tau) - \theta_{\text{prev}}] \exp\{-[\tau - \tau_{\text{prev}}](\alpha(\tau))\} \]  
(17c)

\( \beta(\tau) \) is obtained from the experimentally measured specific oxygen consumption rate for cats. Figure 5 shows experimental data for the fever of cats, obtained by Sachdeva et al. In order to predict fever, the \( \beta(\tau) \) was calculated using the experimental data on specific oxygen consumption. It is apparent that a very good agreement has been obtained between the model and data during heat-up period. Since \( \beta(\tau) \) was as high as 1.5, while \( \theta \) maximum is on the order of 1.2 to 1.3 at \( \tau = 250 \) min, Eq. (15a) predicts that \( \theta / \theta_{\text{prev}} \tau > 0 \), temperature continues to increase if \( \alpha = 1 \). No data for \( \tau \) or \( \beta(\tau) \) was available after 260 min. The oxygen consumption rate was artificially lowered to the normal level for the modeling results which then led to decreased temperature. Hence, \( \beta(\tau) \) was set at 1 after \( \tau = 260 \) min; subsequently the model predicts the temperature to decrease. Typically, the model’s predicted temperature peak is higher than the experimental data. This can be explained by the assumption that all energy produced from fuel oxidation contributed to heat production. By letting \( \alpha \) decrease linearly from 1 to 0.95 within the first 30 minutes during which shivering was observed, the temperature rise was observed to be rapid. This is expected since heat-loss is expected to decrease. Again, letting \( \alpha = 1 \) at \( \tau = 250 \) min which simulates the restoration of blood flow, the temperature vs time was calculated for \( \tau > 260 \) min. Further, the experimental data within the limited range seems to suggest that the oxygen consumption remained above normal even after 2 hours, which indicated a higher than normal temperature.

SUMMARY

From nasal exhaust gas analysis, the composition of the fuel can be determined and a relationship between the core temperature and the rate of oxygen consumption can be established. By applying the first law of thermodynamics to a human or animal body, the net rate of heat-loss can be estimated. While the predicted fever model follows the experimental data on cats, the model is not exact. The extent of the effect of ATP and a varying blood flow to the skin during fever is yet to be explored.

ACKNOWLEDGMENTS

The authors wish to thank Ms. Jothi of The University of Texas Health Science Center, San Antonio for educating us on the metabolic activities of the human body.

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Fig. 3: Non-Dimensional Temperature vs Time

Fig. 4: Temperature vs Time for Fever Model, $\alpha=1$: Normal heat loss rate increased metabolic rate, $\beta = 1.75$. $\alpha = 0.57$: Normal Metabolic Rate But Reduced Heat-Loss, $\beta = 1$.

Fig. 5: Temperature vs Time: Comparison of Model Results with Experimental data on cat; Model Results with and without reduced Blood Flow (Red Bl)