OPTIMAL DESIGN OF THERMAL SYSTEMS AND COMPONENTS

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OPTIMIZATION OF THE STORAGE PROCESS
FOR A COOL THERMAL STORAGE SYSTEM

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ABSTRACT

The storage process for a static-water ice-on-coil cool thermal storage system is difficult to model analytically, based on the dynamic behavior of ice production. Systems that utilize a vapor-compression cycle, with the tank acting as an evaporator, further complicate an analytical model due to the two-phase heat transfer throughout the storage tank. This analysis presents a simplified model of the storage process for a static-water ice-on-coil storage tank acting as an evaporator in a vapor-compression cycle.

Specifically, the storage process is optimized by minimizing the amount of compressor work required to freeze water at 0°C. Optimization variables are refrigerant evaporating temperatures and tank heat exchanger sizing. The dynamics of ice production and two-phase heat transfer are simplified by assuming the overall heat transfer coefficient remains constant throughout the storage process. An average value for the overall heat transfer coefficient may be substituted and still provide useful results. A second law analysis utilizing the irreversibility developed during cool storage, is also presented. The model is then used in side-by-side comparisons of compressor work, tank heat exchanger efficiency, and irreversibility, as functions of evaporating temperature, for several heat exchanger sizes.

INTRODUCTION

Cool thermal energy storage, CTES, is an effective means of shifting peak demand for electrical energy to more economical off-peak hours. While there is little doubt as to the peak shifting ability of CTES, concern exists as to whether designs are as energy efficient as feasibly possible. This paper presents an optimization method for the storage process for a CTES system utilizing a static water ice-on-coil tank, which acts as an evaporator in a refrigerant vapor-compression cycle.

Based on the standard vapor-compression cycle, compressor work is optimized through the fine tuning of heat exchanger sizing and refrigerant evaporating temperatures. Evaporating temperature, in this context, refers to the temperature at which the refrigerant is evaporated as it flows through the tank during the freezing process. Heat exchanger size is included in a dimensional parameter referred to as the heat exchanger number, or HEN. The HEN is simply the ratio of the overall heat transfer coefficient, UA, to the mass flow rate of the refrigerant, \( \dot{m}_{\text{ref}} \), and has units of kJ/\( \text{kg-K} \). Both evaporating temperature and HEN are varied to determine the optimal operating conditions corresponding to minimized compressor work.

A simplified second law analysis is also performed to determine the quality of the thermodynamic storage process. Specifically examined are irreversibilities developed from the heat transfer occurring within the storage tank and from the irreversible expansion of refrigerant in the heat exchanger. From examination of the behavior of these components a better understanding of the storage process is obtained.

Previous work in the analysis of a thermal storage system has been performed by Adrian Bejan (1978). Bejan's work concentrated on a heat storage vessel which was charged with hot steam. Bejan found storage time and charging temperature to play significant roles in minimizing the irreversibility developed during the storage process. An attempt to extend Bejan's work to an ice storage system ran into several barriers, including the modeling of two phase flow along with the transient behavior of ice production. The complexity of the process also prevented the development of a non-dimensional
solution. A steady state model is presented here, which ignores the dynamics of the process, yet still offers useful design techniques.

**ANALYSIS**

The simplified model of the storage tank consists of two control volumes and is shown in Figure 1. Control Volume 1, (CV 1), encompasses the entire refrigerant circuit, including the compressor, condenser and tank heat exchanger. Energy is transferred as heat from the solidifying water, to the control volume, through the evaporator heat exchanger located within the tank. Energy is removed as heat, through the heat exchanger located within the condensing unit. The compressor is also located within the condensing unit and is used to transfer energy, as work, to the refrigerant.

**FIGURE 1 SIMPLIFIED MODEL OF ICE-ON-COIL STORAGE SYSTEM, STORAGE MODE**

On a per mass of water basis, the amount of energy stored latently is significantly greater than the amount of energy stored sensibly. For this reason, this analysis will concentrate on latent storage at 273K, (0°C). Figure 2 further illustrates the transfers of energy as heat and work to the refrigerant cycle along with the temperatures at the points of heat transfer. This heat transfer model is the basis of the compressor work analysis to follow.

Control Volume 2, (CV2), contains the same components as the first while also including the water within the storage tank. By including the tank within the control volume, the heat transfer component at the evaporator will be replaced with an energy storage term. This control volume will be used in the second law analysis to examine components of irreversibility developed during the storage process.

**FIGURE 2 ENERGY DIAGRAM FOR STORAGE PROCESS**

**Thermodynamic Cycle**

The ideal thermodynamic cycle followed by the refrigerant during the charging process is shown in Figure 3 (A), as a temperature-entropy diagram, and in Figure 3 (B), as a pressure-enthalpy diagram. The properties of R-22 are used to illustrate the technique, however the model presented is refrigerant independent. The assumptions made for the actual process are as follows:

1. Isentropic vapor compression from saturated state to condenser pressure
2. Internally reversible heat rejection at constant pressure, not necessarily isothermal
3. Irreversible expansion at constant enthalpy, from saturated liquid to evaporator pressure
4. Internally reversible heat addition at constant pressure

**FIGURE 3 (A) TEMPERATURE-ENTROPY DIAGRAM (B) PRESSURE-ENTHALPY DIAGRAM FOR R-22 DURING STORAGE PROCESS**

With these definitions in mind, the thermodynamic expressions for energy transfers, and corresponding assumptions, can be derived. The assumptions are as follows:

Subscripts including 'c' or 'e' imply condensing or evaporating conditions, respectively.
1) Isentropic compression from 4 to 1,

\[ T_{1,\text{ref}} = T_{e,\text{ref}} \left( \frac{P_{2,\text{ref}}}{P_{1,\text{ref}}} \right)^{\frac{k-1}{k}}, \quad k=1.22 \tag{1} \]

2) Condenser pressure, \( P_{e,\text{ref}} = P_{2,\text{ref}} \) \( T_{2,\text{ref}} \)

3) Isenthalpic irreversible expansion from 2 to 3, \( h_3 = h_{2,f} \) \( \tag{3} \)

The transfers of energy to the refrigerant with heat and work are, on a rate basis:

\[ \dot{Q}_{\text{evap}} = \dot{m}_{\text{ref}} (h_4 - h_3) \tag{4} \]

\[ \dot{W}_{\text{comp}} = \dot{m}_{\text{ref}} (h_1 - h_4) \tag{5} \]

**Tank Heat Transfer Considerations**

With the basic concepts outlined for the refrigeration cycle, the process for the transfer of energy as heat from the tank to the evaporating refrigerant must now be considered in more detail.

The expression for the energy removed from the tank water, through the heat exchanger, \( \dot{Q}_{\text{tank}} \), is:

\[ \dot{Q}_{\text{tank}} = UA (T_{\text{water}} - T_{e,\text{ref}}) \tag{6} \]

The overall heat transfer coefficient, \( UA \), is the conductance for heat transfer from the saturated ice-water experiencing fusion, through ice formed on the tubes, to the refrigerant flowing within the tubes. The heat transfer circuit for this process is given in Figure 4.

**FIGURE 4 HEAT TRANSFER CIRCUIT FOR TANK HEAT EXCHANGER, DURING FREEZING PROCESS (University of Texas, 1993)**

If the temperature of the water is assumed to be equivalent to the temperature at the edge of the ice layer, the expression for the overall heat transfer coefficient becomes:

\[ UA = \frac{1}{R_{\text{ref}} + R_{\text{tube}} + R_{\text{ice}}} \tag{7} \]

The dynamics of the storage process are initially evident in Figure 4. The ice component of the heat transfer resistance acts as a potentiometer, varying heat transfer capability with ice growth. This variable along with two phase heat transfer hinders the development of a simplistic expression for \( UA \). This problem will be approached further in the development of an expression for compressor work.

The maximum amount of energy that can be transferred from the tank to the evaporating refrigerant, occurs when \( \dot{Q}_{\text{tank}} \) is equal to \( \dot{Q}_{\text{evap}} \). However, if the tank heat exchanger is not large enough to remove the potential amount of energy, \( \dot{Q}_{\text{evap}} \), this energy must be removed by other means so that the refrigerant may enter the compressor with a quality of 1.0. In this application, the energy removed, \( \dot{Q}_{\text{waste}} \), is not utilized in a manner useful to the rest of the cycle.

Figure 5 illustrates this heat transfer process for the vapor-compression cycle, on the P-h diagram. For this process, the tank heat exchanger is most effective when \( \dot{Q}_{\text{evap}} \) is equal to \( \dot{Q}_{\text{tank}} \), that is when the equality holds in the relation:

\[ UA (T_{\text{water}} - T_{e,\text{ref}}) \leq \dot{m}_{\text{ref}} (h_4 - h_3) \tag{8} \]

The fraction of energy removed from the tank to the energy absorbed by the evaporating refrigerant during the cycle, is:

\[ \frac{\dot{Q}_{\text{tank}}}{\dot{Q}_{\text{evap}}} = \frac{UA (T_{\text{water}} - T_{e,\text{ref}})}{\dot{m}_{\text{ref}} (h_4 - h_3)} = \frac{HEN (T_{\text{water}} - T_{e,\text{ref}})}{(h_4 - h_3)} \tag{9} \]

A heat exchanger design in which this ratio is greater than one is oversized. Thus, the limit on the heat exchanger number, HEN, is defined as:

\[ HEN = \frac{UA}{\dot{m}_{\text{ref}}} \leq \frac{(h_4 - h_3)}{(T_{\text{water}} - T_{e,\text{ref}})} \tag{10} \]

**FIGURE 5 HEAT TRANSFER PROCESS FROM TANK TO EVAPORATOR**

With the heat exchanger component of the model established, the next step in the analysis is to determine the amount of compressor work required to freeze a given amount of ice, for several HENs, over a range of refrigerant evaporating temperatures.

**Compressor Work Considerations**

The compressor work rate was determined for the charging process in equation 5. The amount of work required is the work rate, which will be assumed as constant, integrated over the time to freeze,

\[ W_{\text{comp}} = \int_{t_0}^{t} \dot{W}_{\text{comp}} \, dt \tag{11} \]
An expression for incremental freezing time, \(dt\), is found by dividing the incremental amount of heat transferred from the freezing water by the rate of heat transfer. The incremental amount of heat transferred is simply the latent heat of fusion of water, LHF, multiplied by the incremental mass of ice produced. Therefore,

\[
dt = \frac{\Delta Q_{\text{fusion}}}{\Delta Q_{\text{bank}}} = \frac{\text{LHF}}{(T_{\text{water}} - T_{\text{ref}})} \left( \frac{M_{\text{ice}}}{UA} \right)
\]

(12)

where,

- LHF = Latent heat of fusion of water, 334.3 kJ/kg at 273K.

The dynamic behavior of ice production alone with two phase heat transfer appears again in equation 12. An analytical model addressing both two phase flow and the moving boundary conditions was found to be too complex to be incorporated into this model. An accurate numerical analysis would require simultaneously solving multiple equations linking control volumes along the refrigerant path through the tank, with the overall amount of heat transfer from the tank to the refrigerant. A computerized numerical model for this type of storage system has been developed and was undergoing verification at the time of this research (Peterson, 1993).

For the sake of simplicity, this analysis will proceed to determine required compressor work, with the following liberal assumptions regarding storage size:

1) Water is at 273K, or 0°C throughout the latent storage process.
2) The initial state of water is liquid and the final state is solid.
3) The overall heat transfer coefficient, \(UA\) remains constant throughout the process.

Integrating equation 12, the relation for the amount of time required to freeze a given amount of ice becomes,

\[
t = \frac{\text{LHF} \times M_{\text{ice, total}}}{UA(T_{\text{water}} - T_{\text{ref}})}
\]

(13)

where,

- \(M_{\text{ice, total}}\) = Total mass of ice produced.

Obviously, this expression is based on broad assumptions, however a simplified analysis is often the best way to begin the study of a difficult problem.

Assuming that the rate of compressor work remains constant throughout the storage process, the amount of work required per mass of ice produced is derived by simply multiplying the work rate by the storage time. For the storage process, from equations 5 and 13:

\[
\frac{W_{\text{comp}}}{M_{\text{ice, total}}} = \frac{\text{LHF} \times (h_t - h_q)}{(T_{\text{water}} - T_{\text{ref}})} \times \left( \frac{1}{T_{\text{ref}}} - \frac{1}{T_{\text{water}}} \right) \times \text{HEN}
\]

(14)

**Irreversibility Considerations**

A means of interpreting the quality of the thermodynamic process is through the use of the second law. By applying the second law to Control Volume 2 an expression for the work lost from entropy generation, or irreversibility, may be developed. In reference to Figure 1, the steady state energy balance for Control Volume 2 is:

\[
W_{\text{comp}} = Q_{\text{cond}} + \Delta U_{\text{water}} - Q_{\text{waste}}
\]

(15)

Similarly, the steady state entropy balance is:

\[
\sigma = \Delta S_{\text{water}} + \frac{Q_{\text{cond}}}{T_{\text{water}}} - \frac{Q_{\text{waste}}}{T_{\text{air}}}
\]

(16)

where \(\sigma\) is the entropy produced during the storage process.

Within Control Volume 2 there are two components contributing to total irreversibility; the heat transfer from the tank to the evaporating refrigerant and the expansion of the refrigerant across the expansion valve. (Recall that the compression of the refrigerant was assumed to be isentropic.) Irreversibility is interpreted as the increase in required compressor work resulting from the production of entropy within the control volume. For Control Volume 2, the total irreversibility is simply:

\[
I = T_{\text{ref}} \sigma
\]

(17)

Combining equations 15, 16 and 17, the total irreversibility developed, per mass of ice produced, is given in equation 18:

\[
\frac{I_{\text{total}}}{M_{\text{ice, total}}} = -\text{LHF} \times \left( \frac{T_{c, air} - 1}{T_{\text{water}}} \right) + \frac{W_{\text{comp}}}{M_{\text{ice, total}}}
\]

(18)

The assumptions made in deriving equation 18 are:

1) The entropy generated from the volume expansion of water is negligible.
2) Initial state of water is liquid and the final state is solid, both at 273K.

By distinguishing between each of the irreversibility components, a better understanding of the thermodynamic process can be achieved. The component of irreversibility developed by the tank heat transfer is found by applying an approach similar to that used to determine compressor work, across the surface of the tank heat exchanger. That is, the expression for the rate of irreversibility developed via tank heat transfer is:

\[
\frac{I_{\text{heat}}}{M_{\text{ice, total}}} = m_{\text{ref}} \times T_{c, air} \times \text{HEN} \times (T_{\text{water}} - T_{\text{ref}}) \times \left( \frac{1}{T_{\text{ref}}} - \frac{1}{T_{\text{water}}} \right)
\]

(19)

The total heat transfer irreversibility produced is determined by integrating equation 19 over the complete storage time. As was done with compressor work, the expression for time from equation 13 and corresponding assumptions are used in the integration. The resulting expression becomes:

\[
\frac{I_{\text{heat}}}{M_{\text{ice, total}}} = T_{c, air} \times \text{LHF} \times \left( \frac{1}{T_{\text{ref}}} - \frac{1}{T_{\text{water}}} \right)
\]

(20)

Finally, the irreversibility developed by the expanding refrigerant during the storage process is simply the difference
between the total irreversibility and the heat transfer irreversibility:

\[
\frac{I_{\text{expand}}}{M_{\text{ice,total}}} = \frac{I_{\text{total}}}{M_{\text{ice,total}}} - \frac{I_{\text{heat}}}{M_{\text{ice,total}}}
\]

(21).

RESULTS OF ANALYSIS

Figure 6 shows the behavior of the ratio of \(Q_{\text{total}}\) to \(Q_{\text{evap}}\) as a function of evaporating temperature, for HEN's of 2.0 to 12.0. For heat exchangers with large HEN's, a heat transfer ratio of 1.0 is reached at temperatures closer to that of the freezing water at 273K. Recall that the maximum value of the HEN is dependent on the evaporating temperature.

![Figure 6: Heat Transfer Ratios as a Function of Evaporating Temperature](image)

**Figure 6: Heat Transfer Ratios as a Function of Evaporating Temperature**

Figure 7 looks at the heat transfer ratios as functions of HEN, for the range of evaporating temperatures from 200K to 270K. The maximum value of the HEN is reached for a given temperature when these curves level off at 1.0. From this perspective, it is clear that as the evaporating temperature nears that of the water, in order to achieve a heat transfer ratio of 1.0, the size of the heat exchanger approaches an 'infinite' value.

Figures 8 and 9 show the effect on compressor work of varying evaporating temperature and HEN, respectively. In Figure 8 specifically, most of the heat exchangers experience minimas in compressor work over the range of temperatures examined. The decrease in work as temperatures fall below 270K is due to the enhanced heat transfer rate that occurs with lower evaporating temperature. This behavior is indicative of a dominant time factor. Below the temperature where the minima occurs, compressor work increases as the benefits of increased heat transfer potential no longer dominates the process efficiency. Beyond the minima, time is no longer dominant.

![Figure 7: Heat Transfer Ratios as a Function of HEN](image)

**Figure 7: Heat Transfer Ratios as a Function of HEN**

![Figure 8: Compressor Work Required per KG of Ice Produced, with Evaporating Temperature](image)

**Figure 8: Compressor Work Required per KG of Ice Produced, with Evaporating Temperature**

![Figure 9: Compressor Work Required per KG of Ice Produced with HEN](image)

**Figure 9: Compressor Work Required per KG of Ice Produced with HEN**

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For the purpose of clarity, some graphs will utilize markers to better illustrate the curves and should not be interpreted as data points.
For the heat exchangers of Figure 8, the compressor work minima may not correspond to the evaporating temperature at which the heat transfer ratio is one, (Figure 6). Note that the points represent compressor work at discrete evaporating temperatures, with 5K and 10K intervals. Thus actual minima may not be evident at these temperature intervals.

Figure 10 illustrates the optimized HEN's required to achieve minimized compressor work, for evaporating temperatures at 10K intervals. The heat exchangers of Figure 10 have heat transfer ratios of 1.0 and thus have no need for waste heat removal. As shown in Figure 9, for a given evaporating temperature, heat exchangers sized below the optimum require more compressor work due to the reduction in the heat transfer surface. Those heat exchangers sized larger than the optimum will only increase equipment costs while having little effect on reducing compressor work.

![Figure 10](image)

**FIGURE 10 OPTIMAL HEN REQUIRED TO OBTAIN MINIMAL COMPRESSOR WORK FOR SEVERAL EVAPORATING TEMPERATURES**

Figure 11 shows the components of the total irreversibility developed during the storage process, per mass of ice produced. Recall the sources of these components were the irreversibilities developed due to tank heat transfer and that due to refrigerant expansion, respectively. Heat transfer irreversibility is dependent mainly on evaporating temperature since the effect of heat exchanger size was eliminated from equation 20.

Expansion irreversibility is affected somewhat by heat exchanger size, however the most dramatic response seems to be towards evaporating temperature. At temperatures above 240K, expansion irreversibility begins to outweigh heat transfer irreversibility as temperature differences across the heat exchanger are minimized. The large jump at temperatures near 270K is contributed to the excess charging time required with higher evaporating temperatures. Observations such as these can prove to be effective design tools, due to the back door approach that the second law analysis provides.

![Figure 11](image)

**FIGURE 11 HEAT TRANSFER AND REFRIGERANT EXPANSION COMPONENTS OF TOTAL IRREVERSIBILITY, PER MASS OF ICE PRODUCED, FOR STORAGE PROCESS**

It is interesting to examine how all of the previously discussed factors relate to each other. Figures 12 (a) through (c) show the effects of evaporating temperature on heat transfer ratio, compressor work and components of irreversibility per mass of ice produced, for HEN's of 4.0, 8.0, and 12.0.

For a design with a HEN of 4.0, the minimum required amount of compressor work of 150.89 kJ per kg of ice produced, occurs at 240K. For the temperatures considered, total irreversibility is also minimized at 240K. At this minimum, over 55% of the total irreversibility of 117.7 kJ/kg-ice is produced by the expansion of the refrigerant.

Similarly, in Figure 12 (b), at 250K the compressor work is minimized at 94.73 kJ/kg-ice for a heat exchanger with a HEN of 8.0. Total irreversibility, at 61.61 kJ/kg-ice, is reduced somewhat from that corresponding to a HEN of 4.0. The reduction in total irreversibility is devoted to the reduction in expansion irreversibility, which now accounts for 45% of the total.

Finally, in Figure 12 (c), for a HEN of 12, compressor work achieves a minima of 68.744 kJ/kg-ice, at the evaporating temperature of 260K. Total irreversibility continues to decrease with HEN, to a value of 35.62 kJ/kg-ice. The contribution to the total irreversibility of the refrigerant expansion is nearly 47%, an increase over the amount seen with a HEN of 8.0.

It is important to note that the total irreversibility may never be reduced to zero. A storage system with negligible irreversibility would function similarly to a system utilizing the Carnot cycle. Referring to equation 18, if the irreversibility term is omitted, the minimal compressor work is 33.12 kJ/kg-ice, regardless of the evaporating temperature. This corresponds to a Carnot cycle operating within Control Volume 1.

Rigorous testing of this method has not been attempted. Proprietary data were obtained from a manufacturers' R&D tests of a similar storage system (Lennox, 1992). This information
was used to verify the method developed. The averaged value for HEN was obtained from equation 9 with knowledge of average operating conditions monitored during the storage process. The method predicted the compressor work required to freeze tank water within 21% of the recorded data. In consideration of the simplicity of the method presented here, this result was encouraging.

CONCLUSIONS
A simple model of a static-water ice-on-coil storage system was used to determine ideal evaporating temperatures and heat exchanger sizing. These optimums were based on minimizing the compressor work required per kg of frozen water. The complexity of modeling ice production called for broad assumptions to be made regarding the storage process. Due to the dynamic behavior of the ice-making process, accurate modeling would require in-depth computer simulation. In this preliminary study, the transient behavior was substituted with a steady state model and tested.

From the results of the steady state model, the complexity of the problem is still evident. Significant parameters such as HEN, evaporating temperature, heat transfer ratio and irreversibility were examined for their effects and/or behavior on compressor work required per mass of ice produced. The effect of time, corresponding to heat transfer potential, was also found to be substantial. It was determined, however, that no single parameter was prevalent in minimizing compressor work. An optimized system is one in which several parameters have been finely tuned.

Further studies should include the dynamics of the storage process. An empirical model incorporating transient ice-building and two-phase refrigerant flow would be useful in the determination of HEN and also for the development of a transient optimization model.
Details of this study and others in ice thermal storage may be
found in the Master's Thesis "Investigations in Cool Thermal
Storage: Storage Process Optimization and Glycol Sensible

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